12 12

AN INVESTIGATION OF THE PERFORMANCE OF A MODIFIED TESLA TURBINE

A THESIS

Presented to

the Faculty of the Division of Graduate Studies

Georgia Institute of Technology

In Partial Fulfillment

of the Requirements for the Degree of

Master of Science in Mechanical Engineering

by

James H. Armstrong

June 1952

AN INVESTIGATION OF THE PERFORMANCE

OF A MODIFIED TESLA TURBINE

Approved:

Date Approved by Chairman april 23, 1952

ACKNOWLEDGMENTS

I wish to express my appreciation to my thesis adviser, Prof. W. A. Hinton, of the School of Mechanical Engineering, for his patient understanding and helpful suggestions during this undertaking. I wish to thank Mr. T. D. Sangster, also of the School of Mechanical Engineering, for his interest and help in setting up the equipment.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

AN INVESTIGATION OF THE PERFORMANCE OF A MODIFIED TESLA TURBINE

I

INTRODUCTION

The conventional type of steam and gas turbine, while being a highly efficient machine, has at least two disadvantages. The first of these disadvantages is the complex nature of the blading, resulting in an extremely high cost. This complexness applies not only to the rotating blades but also to the stationary ones. The other disadvantage is the comparative fragility of the turbine. Not only is the rotor very delicate, but even the stator can be unduly sensitive. Stodola[1] tells of a case where the variation in pressure in one stage of a turbine caused the diaphragm holding one set of stationary blades to deflect an amount sufficient to force it to touch the rotor. The resulting friction destroyed the turbine before it could be stopped. The blading of most turbines is readily damaged by excessive moisture in the entering steam and the cost of separators to prevent this damage is high.

---

[1]Dr. A. Stodola, Steam and Gas Turbines, authorized translation by Dr. Louis C. Loewenstein, McGraw-Hill Book Company, Inc., 1927. Reprinted by Peter Smith, New York, 1945, Volume II, p. 962.

Other disadvantages of the conventional type turbine include difficulty in balancing the rotor, susceptibility to critical vibrations, and lower efficiencies in the smaller sizes.

In the early 1900's Nikola Tesla[2] directed his efforts toward the production of a turbine without the disadvantages mentioned above.

One of Nikola Tesla's better known discoveries is the induction electric motor.[3] His attempt to produce a turbine which could be analogous to the induction motor led to the development of his turbine. Just as the induction motor uses a rotating magnetic field to turn its rotor, so the Tesla turbine uses a rotating steam "field" to turn its rotor.

In 1906 Dr. Tesla completed his first turbine which was driven by compressed air. This turbine had a rotor diameter of six inches. The rotor consisted of eight thin metal disks spaced closely on a small shaft. Several holes were located near the center of each disk to allow the spent gases to pass axially along the shaft and to exhaust near the center of each end. The compressed air (and later steam) was introduced through a nozzle in the turbine case, tangentially to and

---

[2]John J. O'Neill, Prodigal Genius, the Life of Nikola Tesla, Binghamton, N.Y., Vail-Ballou Press, Inc., 1944, p. 218 to 228.

[3]John J. O'Neill, Prodigal Genius, p. 48 to 56.

parallel with the planes of the rotor disks. This compressed air entering at high velocity would pass spirally around and through the rotor disks. This stream of air (or steam) would be slowed down appreciably by the friction between the air and the rotor disks. The torque developed in slowing down the stream of air turned the rotor of the turbine. This small engine developed thirty horsepower at a speed of twenty thousand revolutions per minute.

Tesla built several later models which were larger and slower but developed much more horsepower. Most of these later turbines were driven by steam. All of these were single-stage turbines using free exhaust. The following data[4] were given for a 200 horsepower turbine:

Inlet pressure (saturated steam)	125 lbs. per sq. in. gage
Exhaust pressure	Atmospheric
Enthalpy change	130 B.T.U.
Steam consumption	38 lbs. per H.P. Hr.
Speed	9000 R.P.M.

This turbine had a rotor diameter of 18 inches, was three feet long, two feet wide and two feet high; its weight was 400 lbs.

Tesla's turbines were never developed commercially, apparently because of professional jealousies on the part of other engineers and designers, as well as Tesla's lack of

---

[4]John J. O'Neill, Prodigal Genius, p. 222 to 224.

sufficient funds to finance them himself.

Several outstanding advantages for this type of turbine are at once apparent. Among these advantages are: extreme simplicity of construction, low initial cost, relative compactness and ruggedness. This particular type of turbine can be made to reverse by simply adding nozzles which point in the opposite direction.

Some of the difficulties which were experienced in testing the turbines built by Tesla should be mentioned here. After some of the test runs, when the turbines were disassembled and examined, severe distortion of the rotor disks was noted. Several things which probably contributed to this distortion were the extremely high speed of rotation, the high temperatures experienced at the outer edges of the rotor, and the terrific blast of steam striking the blunt edges of the rotor disks.

The efficiencies of Tesla's turbines were considerably lower than those of the more conventional turbines operating under approximately the same conditions. One obvious cause of this low efficiency was the very small clearance between the rotor and the housing, in some cases less than one-sixty-fourth of an inch. With such small clearances the very friction which he was utilizing to drive his turbine was partially holding it back!

All available descriptions of these turbines indicate no attempt at sharpening the edges of the rotor disks was made. Blunt edge disks directly in front of the steam nozzle would produce considerable turbulence in the steam emerging from the nozzle. This turbulence would, of course, help to decrease the efficiency of the turbine.

It is the object of this investigation to determine, if possible, ways to improve upon Tesla's original design, to build and test what is hoped will be an improved version of the turbine, to analyze the turbine from a theoretical standpoint, and to offer suggestions as to further improvements and investigations. Possible commercial applications will also be discussed.

II

ANALYSIS OF THE CYCLE

The steam-driven Tesla turbine, as modified in this experiment, operates on a Rankine cycle. This cycle is shown below on a Mollier Chart:

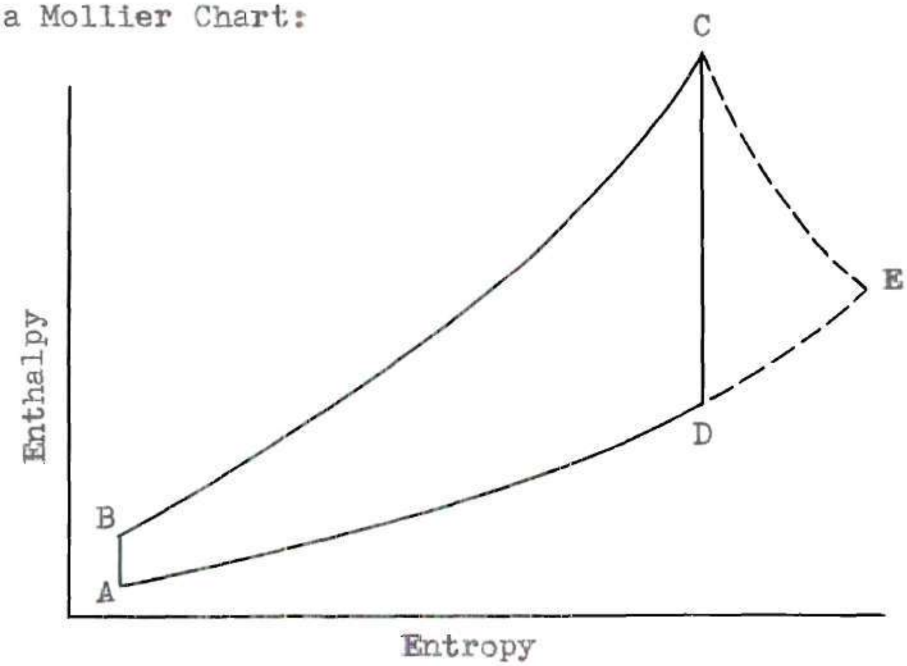


Fig. 1. Mollier Chart of the Rankine Cycle

From A to B work is done on the fluid, in this case water, to bring it to the pressure of the boiler and introduce it into the boiler. From B to C heat is added at constant pressure. From C to D the fluid is expanded through the turbine, in the ideal case at constant entropy by means of a reversible adiabatic expansion. From D to A the fluid rejects heat to the receiver (condenser), and, upon reaching point A, is ready to repeat the cycle. In this ideal case the useful work done by one pound of fluid is equal to the enthalpy change from C to D minus the enthalpy change from A to B. The reversible adiabatic expansion shown from C to D is theoretically impossible of attainment; friction losses, eddy currents, and unavoidable throttling of the fluid reduce the output and increase the entropy so that the actual expansion may be represented by line C E.

It is seen then that to increase the efficiency of the turbine it is necessary for the expansion in the turbine to be as nearly a reversible adiabatic one as possible.

A study of the data from the experimental runs indicates that the Tesla turbine operates as an impulse turbine, that is, nearly all of the expansion of the steam occurs in the nozzle. It is therefore the duty of the rotor simply to absorb as much as possible of the kinetic energy of the steam as it passes through the turbine and to convert this energy into useful work.

An attempt has been made to derive an equation for the theoretical output of a Tesla turbine. This is presented in Appendix III starting on page 55.

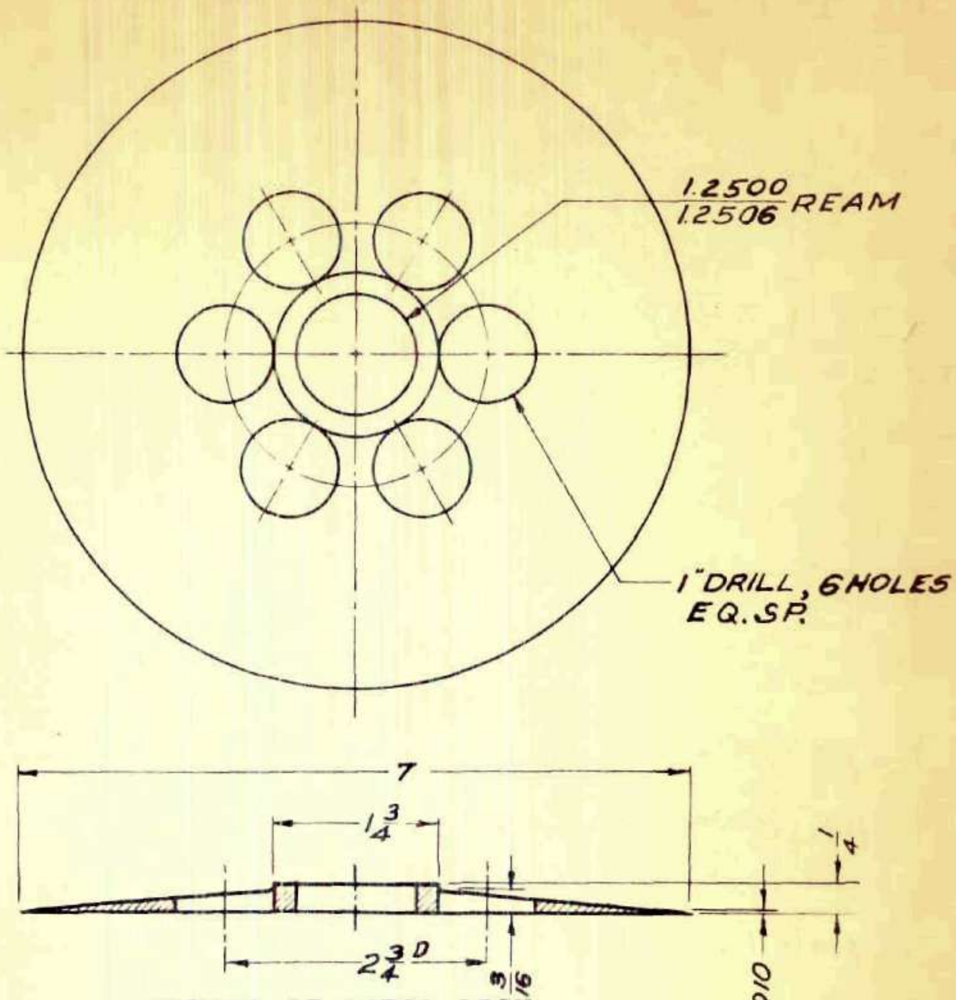
III

DESCRIPTION OF THE TURBINE

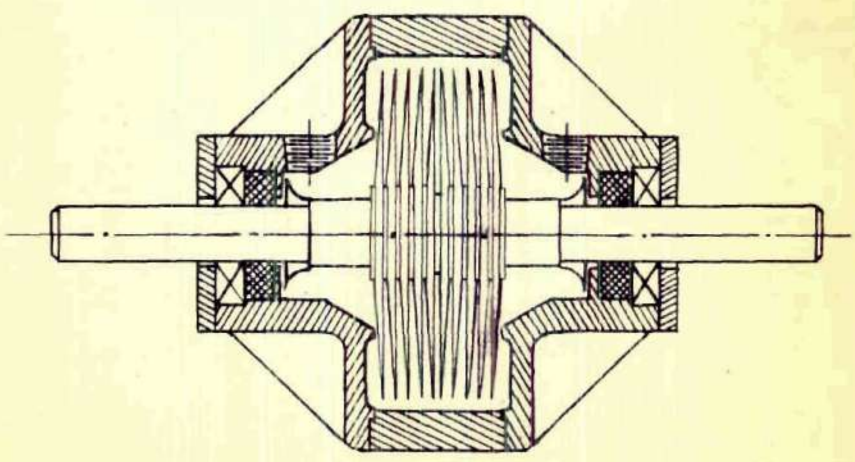
Since the Tesla type of turbine is not available commercially it was necessary to build a working model. It was realized that a suitable means of absorbing and measuring the output would present a real problem so it was decided to design and build a turbine whose maximum output would be no more than four or five horsepower.

This arbitrary limit to the output and size of the turbine introduced another difficulty: namely the fact that small turbines are very inefficient. A comparison of the results of the tests of this turbine with conventional turbines of similar size should at least partially offset this difficulty.

The turbine as built consists of ten rotor disks, each seven inches in diameter and one quarter of an inch thick at the shaft hole. At the place where the disks are mounted, the shaft is one and one quarter inches in diameter. The disks have a force fit on the shaft. The extreme edges of the disks are ten one-thousandths of an inch thick and have a uniform conical taper cut on one side only to a thickness of three-sixteenths of an inch at a distance of one quarter of an inch from the surface of the shaft. A filleted shoulder at that point acts as a spacer for positioning the disks on



DETAIL OF ROTOR DISK



LONGITUDINAL SECTION THROUGH TURBINE

FIG. 2. DETAILS OF TURBINE

assembly with the shaft; see figure 2, page 9.

The disks were made of boiler plate steel. In each disk six one inch holes were drilled equally spaced on a two and three quarter inch diameter hole circle concentric with the shaft. These are the holes which allow the spent steam to pass axially along the shaft and to exhaust from the ends of the turbine.

The housing was made of nickel cast iron in three parts: a central body and two end bells. The clearance between the rotor and the housing averages one quarter of an inch, making the outside diameter of the housing nine inches. The shaft is sixteen inches long, thus we see the turbine is comparable in size to a one horsepower electric motor.

One exhaust port was drilled and tapped in each end bell. Into these were screwed standard three quarter inch pipe nipples for exhaust pipes. For the steam inlet the housing was drilled and tapped so as to admit steam approximately tangentially to the periphery of the rotor disks. For the initial runs a short length of one eighth inch pipe was used as a nozzle. Later several sizes of diverging nozzles were used.

Ball bearings were used on the shaft. A lubrication cup was provided for each bearing; however it was necessary to redesign the oil cups in order to hold against steam pressure. This redesign allows the oil to drip down into the bearings without being held back by steam pressure.

IV

DESCRIPTION OF THE APPARATUS

In addition to the turbine itself several pieces of apparatus were used. Reference to figure 3, page 14, will show more clearly their relationship with each other, as will the photographs shown on pages 15 and 16. The letters refer to the letters shown on the figure, page 14. The letter A indicates the turbine. The other parts of the apparatus are:

B. Prony brake for measuring torque. The moment arm is 21 inches.

C. Howe platform scale used in conjunction with the brake for measuring the torque. The scale has divisions representing 0.01 lb. The weight of the brake arm was counterbalanced by a small piece of galvanized iron placed on the balance pan so that the scale tare was zero. The power output was computed by the formula:

$$HP = \frac{R.P.M \times \text{Scale reading}}{3000}$$

D. A Metron tachometer for measuring the speed of the turbine. The highest scale range, which was the only one used, reads from 0 to 10,000 r.p.m. The scale divisions on this range represent 250 r.p.m. However, it was possible to interpolate to the nearest 50 r.p.m.

E. Steam supply.

F. One-half inch Globe valve used as a throttle to control steam flow to the turbine.

G. Calibrated pressure gage to measure entering steam pressure. This gage has divisions representing 2 pounds per square inch.

H. Throttling calorimeter to determine the quality of the steam.

I. Globe valve to control flow of steam through the calorimeter and to act as the throttling device for the calorimeter.

J. Thermometer, reading from $0^{\circ}F$ to $400^{\circ}F$ in $1^{\circ}F$ divisions, for measuring the temperature of the steam in the calorimeter.

K. A small drain pipe from the bottom of the turbine housing.

L. Calibrated pressure gage for measuring pressure inside the housing. These readings were taken mainly for comparison purposes.

M. Globe valve for draining the turbine housing. This valve is closed during runs. (Not shown in photograph.)

N. Exhaust line.

O. Globe valve to control back pressure; this valve was kept completely open during all runs.

P. Water cooled condenser. The cooling water was not measured, but it was regulated so that a temperature rise of about $15^{\circ}F$ during its passage through the condenser was maintained. This temperature rise was checked by means of two $0^{\circ}F$ to $240^{\circ}F$ thermometers but was not recorded.

Q. Calibrated pressure gage to measure exhaust pressure.

R. Thermometer reading from $0^{\circ}F$ to $400^{\circ}F$ in one deg. F. divisions to measure exhaust temperature.

S. Water pot with water for cooling the brake surfaces.

A large platform scale and a barrel for weighing the condensate are not shown in the figures. The scale has divisions of 1 lb. and an adequate capacity for weighing several hundred pounds when the proper balance weights are used.

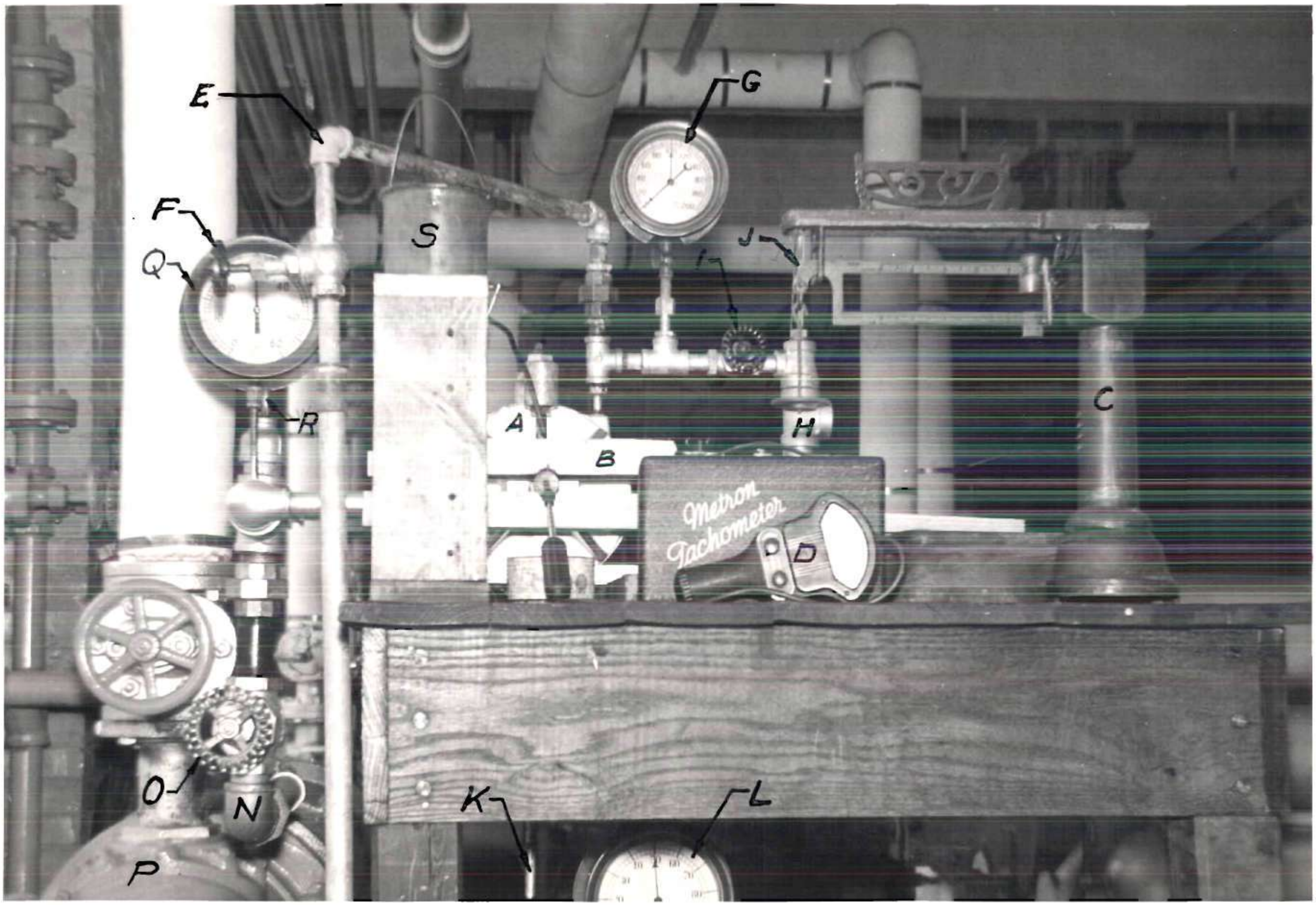


FIGURE 3. PHOTOGRAPH-DIAGRAM OF THE APPARATUS - FRONT VIEW

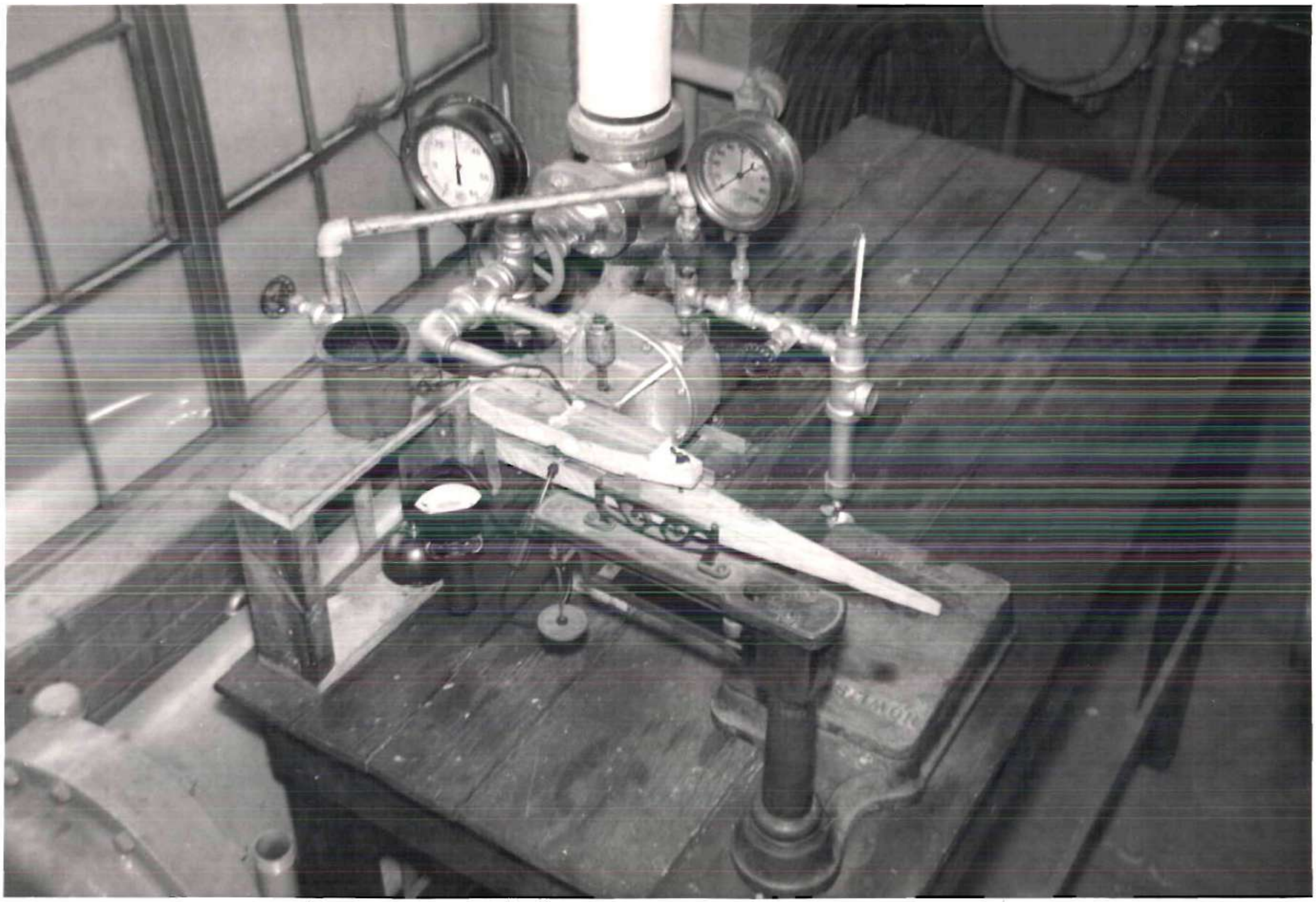


FIGURE 4. PHOTOGRAPH OF THE APPARATUS - TOP VIEW

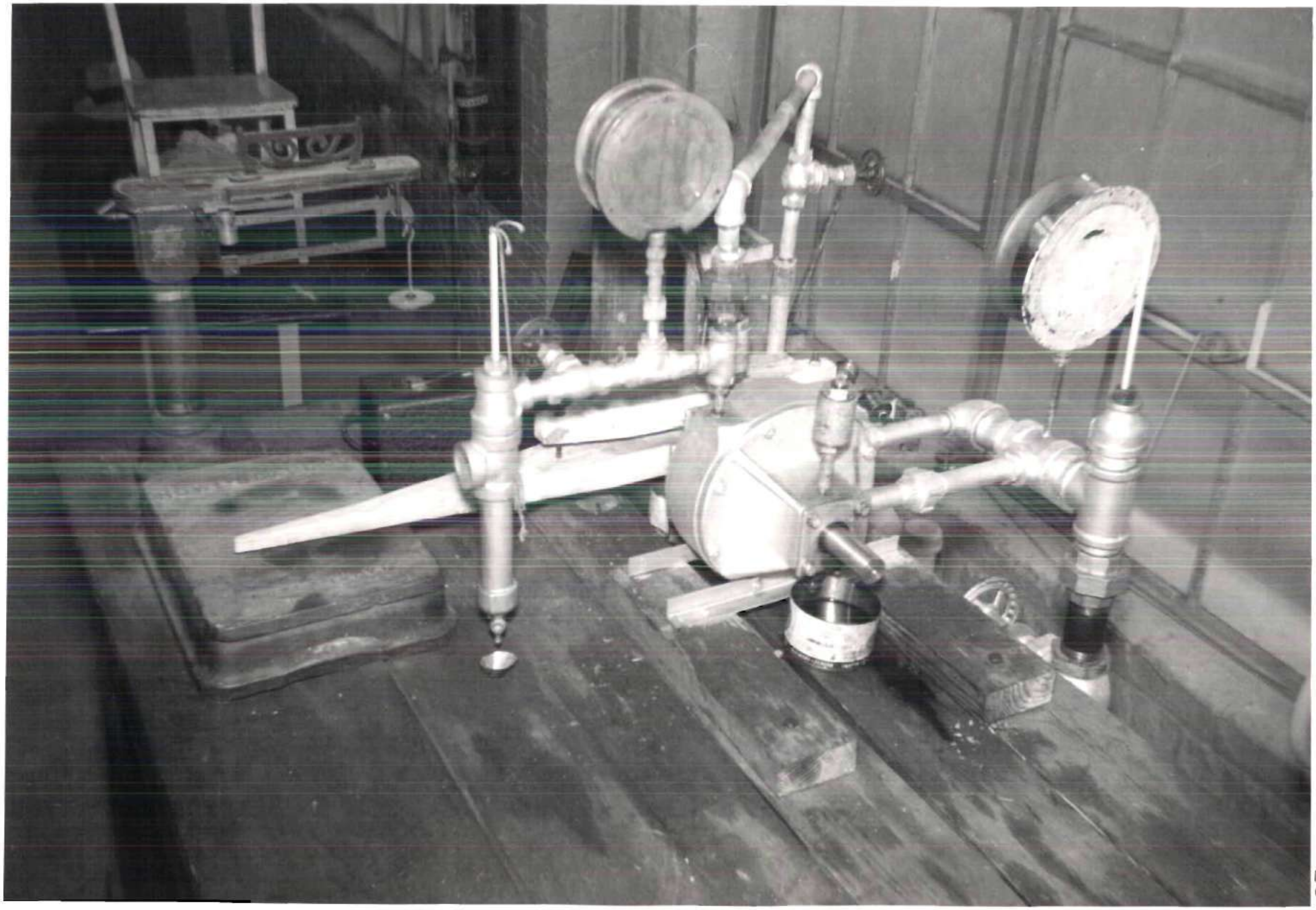


FIGURE 5. PHOTOGRAPH OF THE APPARATUS - REAR VIEW

V

EXPERIMENTAL PROCEDURE

The tests carried out on this turbine may be considered in three groups. The first of these were output tests using a short piece of 1/8 inch pipe for a nozzle. The second group consisted of tests of output using several nozzles with diverging sections. The final group of tests consisted of economy runs at 7,000 r.p.m. using the nozzle which had produced the highest outputs of the turbine during the previous runs.

During all of the tests it was noticed that at the slower speeds it was difficult to maintain constant r.p.m. However, at the higher speeds it was relatively easy to hold the speed constant. This explains the lack of data for the lower speeds.

In the first group of tests, those using the 1/8 inch pipe as a nozzle, the steam pressure, brake scale, and speed were recorded. The inside diameter of an 1/8 inch pipe is 0.269 inches, and, with a nozzle of this size, the output reached 0.52 horsepower at 6,000 r.p.m. and 100 p.s.i. gage inlet pressure. The results of these runs are shown on pages 21 and 22.

For the second group of runs the first nozzle tried had a 0.125 inch throat and a 15^{o} diverging section. This nozzle was made by filling a standard $\frac{1}{4}$ inch pipe nipple which was $1\frac{1}{2}$ inches long with brass. A 0.125 inch hole was drilled through the brass and the diverging section was reamed to an exit diameter of 5/32 inch. The entrance end of the nozzle was counter-drilled with a 5/16 inch drill to a depth of approximately $\frac{1}{2}$ inch.

The turbine was run using this nozzle. The results obtained are given on page 23, and the curves are given on page 24. It can be seen that this nozzle was much to small for efficient operation. It was just about large enough to turn the turbine without external load. With the brake disconnected and 110 lbs. gage inlet pressure the maximum speed was only 5,500 r.p.m.

The next diverging nozzle which was tried had a throat diameter of 0.188 inches and the same type of diverging section as the nozzle just described. It was made by drilling the throat with a 3/16 inch drill and reaming the diverging portion to an exit diameter of 5/16 inch.

The turbine ran much better when this nozzle was used than when the smaller diverging nozzle was used. The results of these runs are given on page 25, and the curves are shown on page 26. With the higher inlet pressures the power developed was in some cases almost one half horsepower. The output, however, was still less than the output obtained with the 0.269 non-diverging nozzle (1/8 inch standard pipe nipple).

The next nozzle tried was a diverging nozzle with a 0.269 inch throat and a seven degree diverging section. The exit diameter was 7/16 inch.

The results of the runs made using this larger diverging nozzle are shown in the chart on page 27 and in the curves on page 28. Even while the runs were being carried on it was evident that the turbine was performing noticeably better than during any previous runs. The maximum output developed during these runs was 1.11 H.P. at 8300 r.p.m. and 118 p.s.i. gage inlet pressure.

The next set of runs performed was to determine the economy and efficiency of this turbine. In the previous runs only output was measured. During these economy runs the quality of the steam entering and the weight of steam used were also measured, as well as inlet pressure and exhaust pressure and temperature. The nozzle used for these economy runs was the one with the 0.269 inch throat and the 7° diverging section. A constant speed of 7000 r.p.m. was maintained. The results are shown in the table on pages 31 and 32, and in the curves on page 33.

VI

EXPERIMENTAL RESULTS

TABLE I. Turbine Characteristics Using 0.269 Inch Straight Nozzle

Steam Pressure Lb./Sq.In. Gage	Speed RPM	Torque Output lb-ft	Brake Horsepower H.P.
30	0	0.350	0
30	1250	0.227	0.054
30	3750	0.175	0.125
30	4900	0.140	0.131
30	5500	0.140	0.147
40	0	0.368	0
40	2000	0.280	0.107
40	3750	0.227	0.163
40	5500	0.175	0.183
40	6100	0.140	0.163
50	0	0.438	0
50	2500	0.350	0.167
50	3500	0.315	0.233
50	5250	0.263	0.262
60	0	0.525	0
60	2500	0.375	0.183
60	4100	0.350	0.274
60	4700	0.315	0.282
60	5400	0.280	0.288
60	6000	0.263	0.300
70	0	0.578	0
70	2400	0.490	0.224
70	4000	0.438	0.333
70	5200	0.420	0.416
70	5350	0.402	0.410
70	5500	0.350	0.367
80	0	0.735	0
80	3300	0.525	0.330
80	3800	0.490	0.355
80	5500	0.420	0.440
100	0	1.050	0
100	3000	0.665	0.380
100	4000	0.630	0.480
100	5000	0.472	0.450
100	6000	0.455	0.520

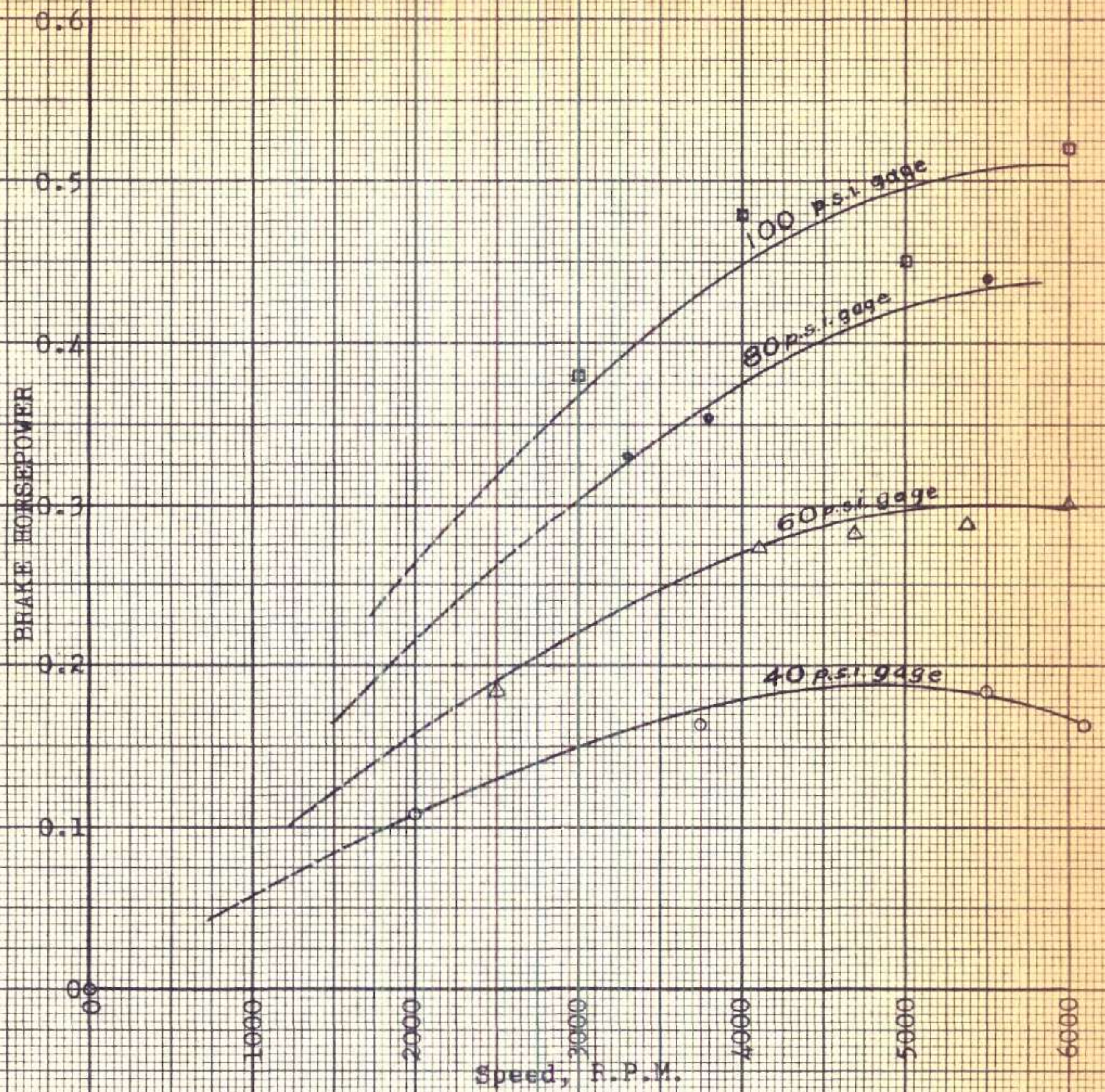


FIGURE 6. TURBINE HORSEPOWER CHARACTERISTICS WITH 0.269 INCH STRAIGHT NOZZLE FOR VARIOUS ENTRANCE PRESSURES

TABLE II. Turbine Characteristics Using 0.125 Inch Diverging Nozzle

Steam Pressure Lb./Sq.In. Gage	Speed RPM	Torque Output lb-ft	Brake Horsepower H.P.
90	0	0.263	0
90	1200	0.035	0.008
90	2600	0.018	0.009
90	4500	0	0
100	0	0.263	0
100	1000	0.070	0.013
100	1500	0.053	0.015
100	2900	0.035	0.019
100	4250	0.018	0.014
100	5300	0	0
110	0	0.175	0
110	900	0.070	0.012
110	2000	0.053	0.020
110	2200	0.035	0.015
110	4500	0.018	0.015
110	5500	0	0
120	0	0.175	0
120	0	0.263	0
120	1400	0.070	0.019
120	2000	0.053	0.020
120	3000	0.044	0.025
120	3200	0.035	0.021
120	4500	0.018	0.015

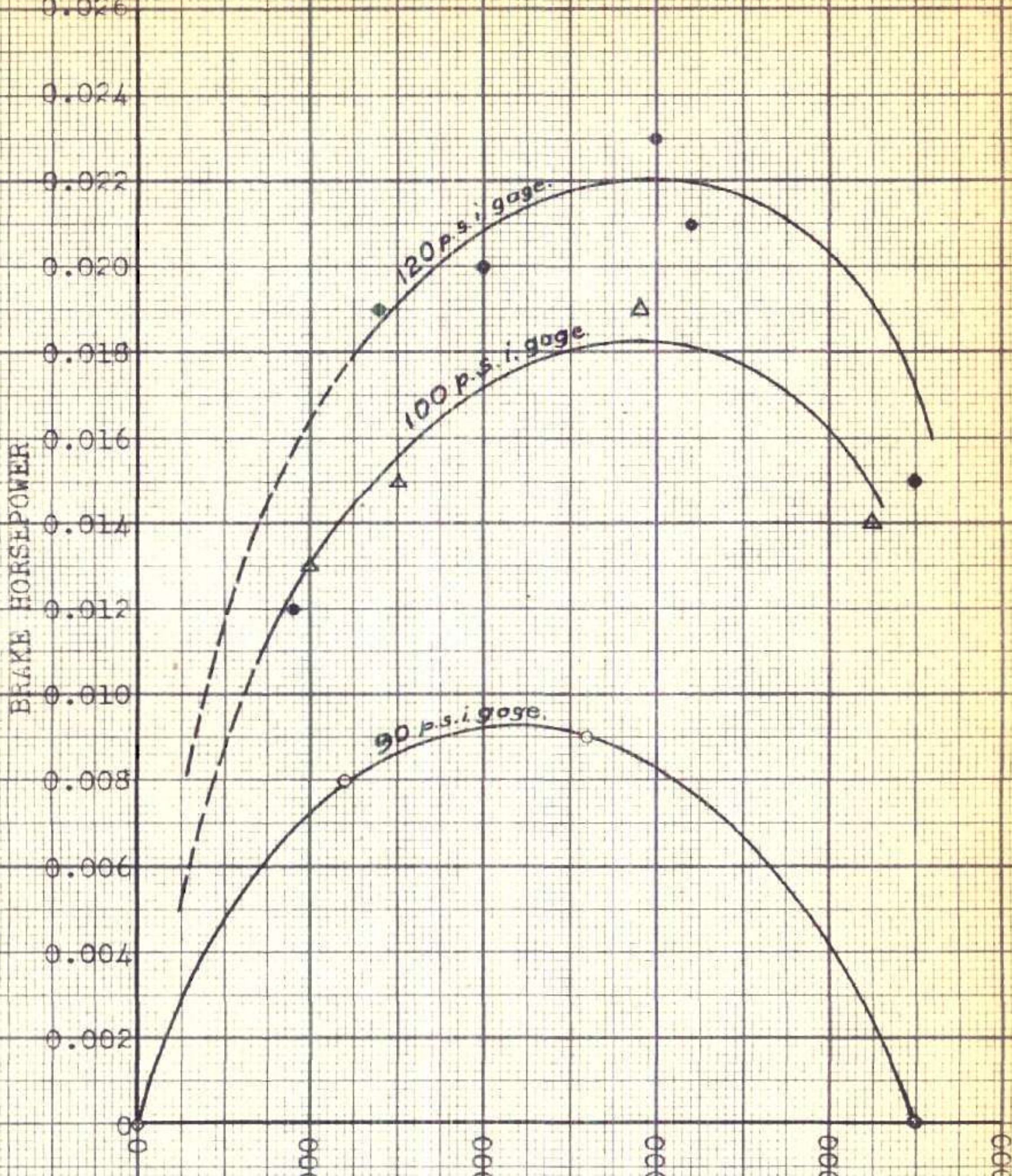


FIGURE 7. TURBINE HORSEPOWER CHARACTERISTICS WITH 0.125 INCH DIVERGING NOZZLE FOR VARIOUS ENTRANCE PRESSURES

TABLE III. Turbine Characteristics Using 0.188 Inch Diverging Nozzle

Steam Pressure Lb./Sq.In. Gage	Speed RPM	Torque Output lb-ft	Brake Horsepower H.P.
40	0	0.158	0
40	1800	0.053	0.018
40	3600	0.017	0.012
40	5500	0	0
60	0	0.210	0
60	100	0.122	0.002
60	3100	0.035	0.021
80	0	0.315	0
80	3200	0.122	0.075
80	5000	0.088	0.083
80	5500	0.053	0.055
80	6000	0.035	0.040
100	0	0.525	0
100	4800	0.210	0.192
100	6500	0.140	0.173
100	6000	0.175	0.200
120	0	0.578	0
120	5000	0.297	0.283
120	5400	0.280	0.288
120	6200	0.263	0.310
120	6800	0.245	0.317
130	0	0.648	0
130	3000	0.437	0.250
130	5500	0.350	0.367
130	6600	0.332	0.418
130	7000	0.297	0.397

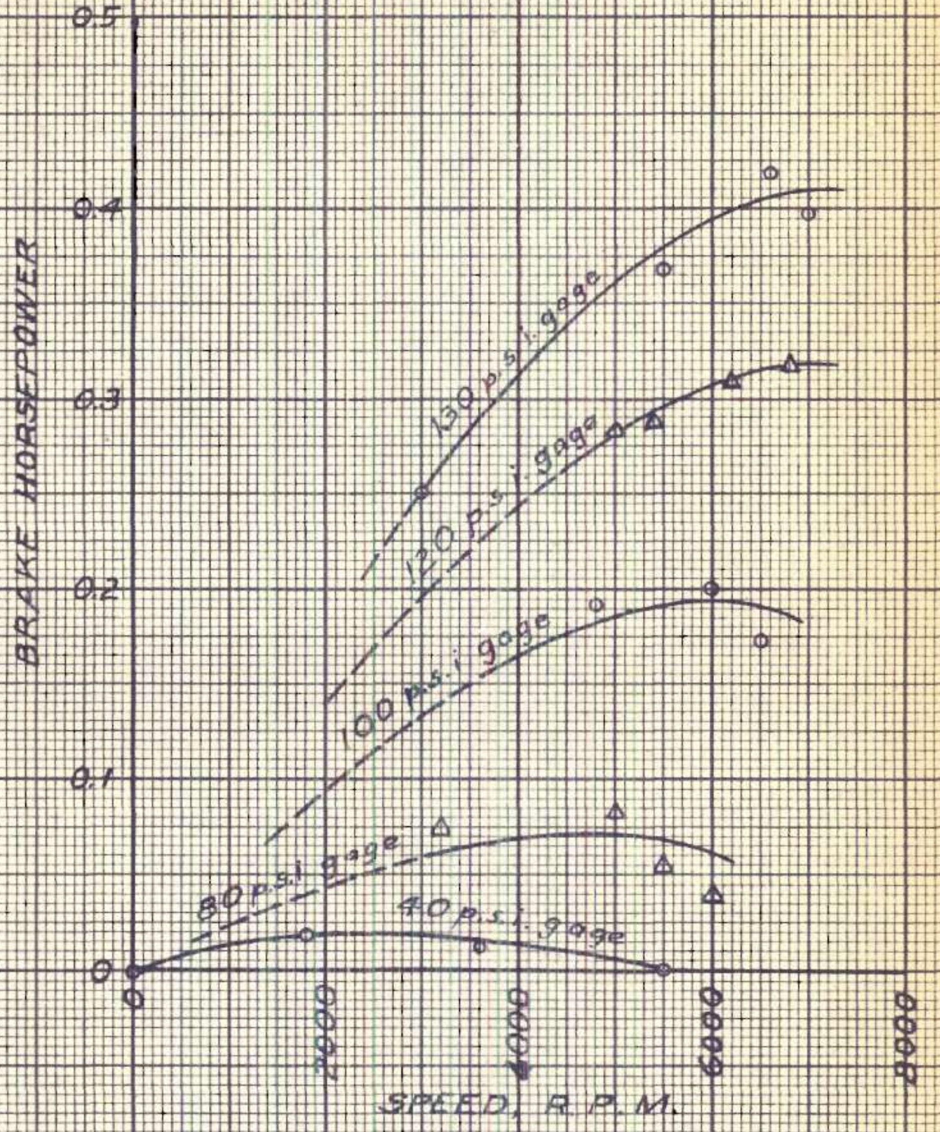


FIGURE 8. TURBINE HORSEPOWER CHARACTERISTICS WITH 0.188 INCH DIVERGING NOZZLE FOR VARIOUS ENTRANCE PRESSURES

TABLE IV. Turbine Characteristics Using
0.269 Inch Diverging Nozzle

Steam Pressure	Speed	Torque Output	Brake Horsepower
Lb./Sq .In. Gage	RPM	lb-ft	H.P.
40	0	0.438	0
40	4000	0.175	0.133
40	4800	0.140	0.128
40	5700	0.105	0.114
40	6500	0.088	0.108
60	0	0.525	0
60	5000	0.350	0.333
60	5300	0.350	0.353
60	5900	0.332	0.374
60	7000	0.263	0.350
80	0	0.700	0
80	4600	0.525	0.460
80	6500	0.437	0.542
100	0	0.954	0
100	3300	0.788	0.495
100	4800	0.700	0.640
100	5000	0.700	0.668
100	6200	0.648	0.765
100	6250	0.612	0.729
100	7500	0.525	0.750
100	7900	0.437	0.659
118	0	1.050	0
118	4500	0.875	0.750
118	5000	0.875	0.833
118	5700	0.875	0.950
118	6200	0.840	0.992
118	6800	0.787	1.020
118	8300	0.700	1.108

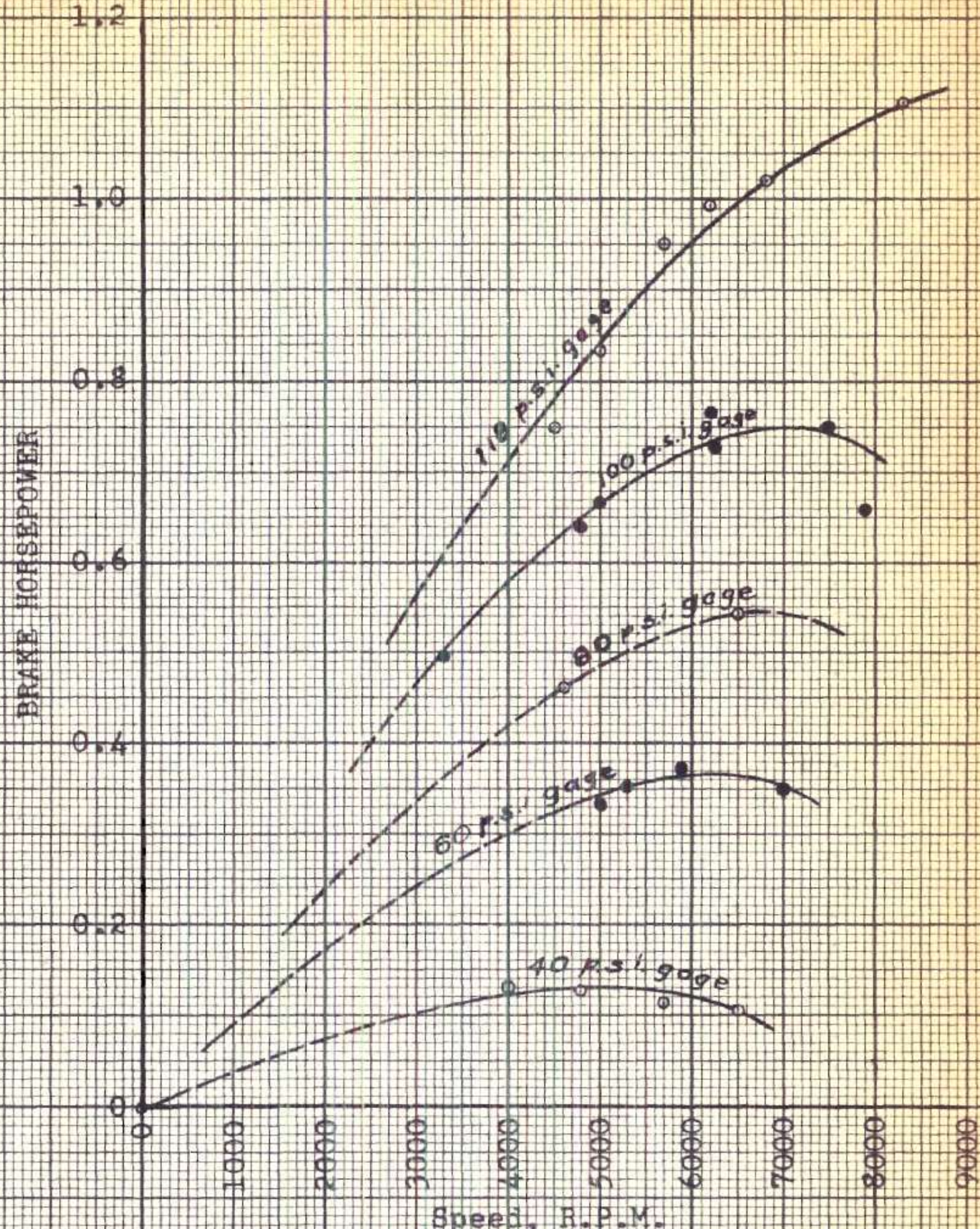


FIGURE 9. TURBINE HORSEPOWER CHARACTERISTICS
WITH 0.269 INCH DIVERGING NOZZLE
FOR VARIOUS ENTRANCE PRESSURES

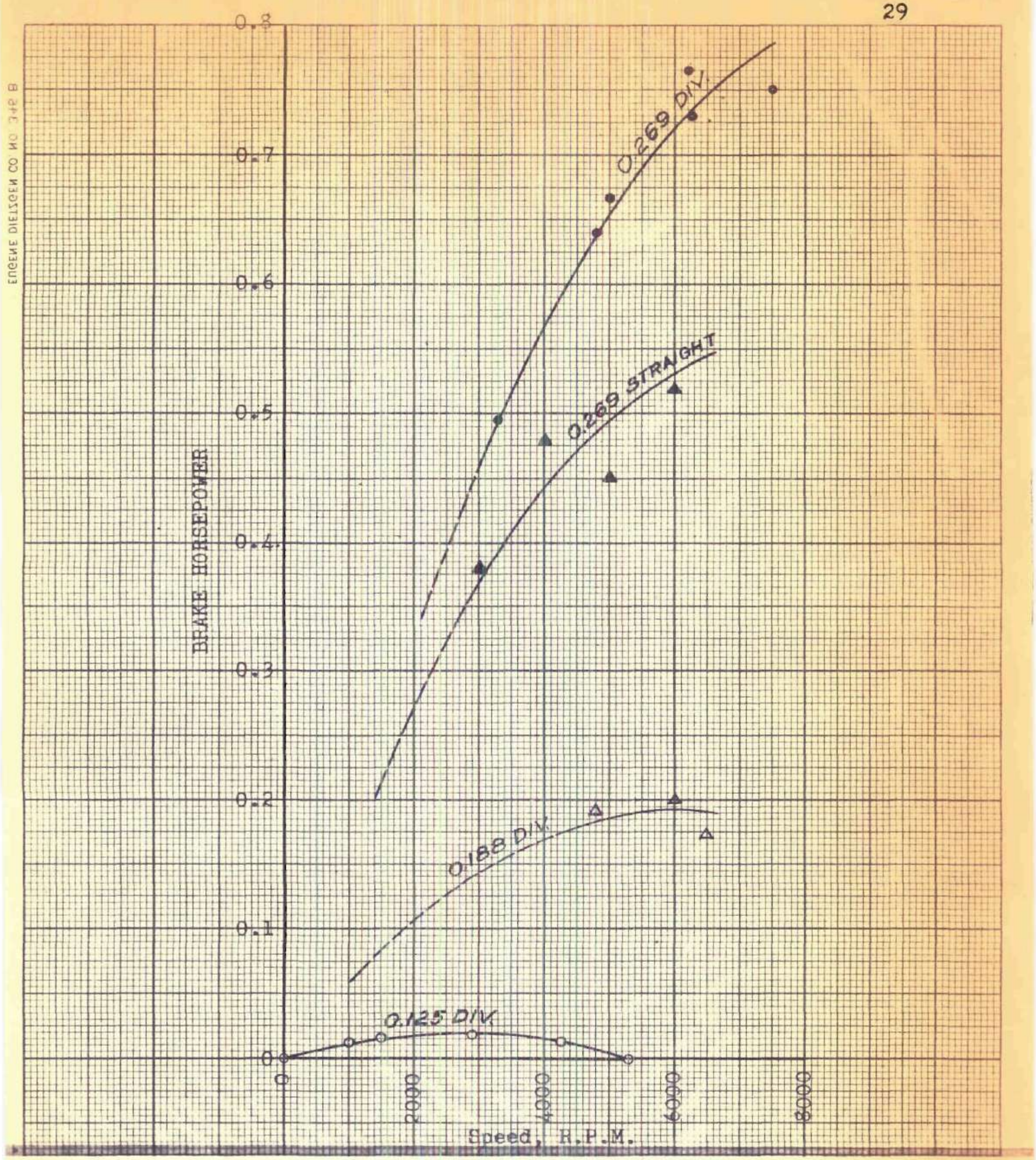


FIGURE 10. COMPARISON OF TURBINE HORSEPOWER AT 100 P.S.I. GAGE INLET PRESSURE USING VARIOUS NOZZLES

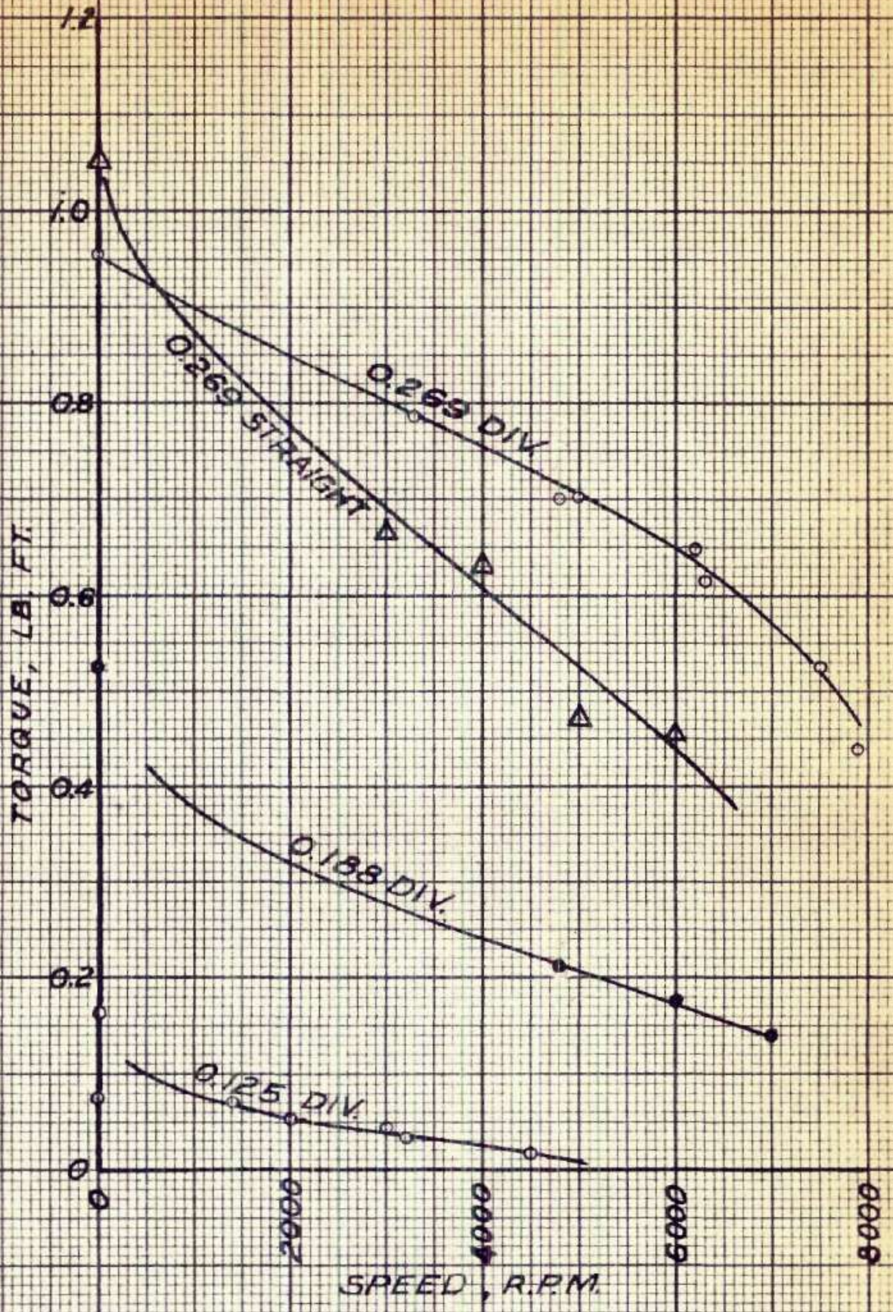


FIGURE 11. COMPARISON OF TURBINE TORQUES AT 100 P.S.I. GAGE INLET PRESSURES USING VARIOUS NOZZLES

TABLE V. Economy Tests on Tesla Turbine With 0.269 Diverging Nozzle

Item					
1	Run No.	1	2	3	4
2	Inlet press.gage, p.s.i.	60	80	90	100
3	Speed, r.p.m.	7000	7000	7000	7000
4	Brake scale, lbs.	0.190	0.236	0.287	0.360
5	Brake horsepower, H.P.	0.443	0.551	0.670	0.840
6	Torque, lbs. ft.	0.333	0.413	0.502	0.630
7	Barometric Press.,In.Hg.	29.24	29.24	29.24	28.88
8	Barometric Press.,p.s.i.	14.36	14.36	14.36	14.20
9	Inlet press., abs.p.s.i.	74.36	94.36	104.36	114.20
10	Calorimeter temp., ^{o}F	230	249.7	256	244
11	Quality of entering Steam, percent	97.4	98.1	98.1	97.3
12	Enthalpy of entering Steam, B.T.U./lb.	1159.0	1169.0	1171.5	1166.0
13	Exhaust pressure, gage p.s.i.	0.8	0.9	1.1	1.1
14	Exhaust pressure, abs. p.s.i.	15.16	15.26	15.46	15.30
15	Exhaust temp., ^{o}F	213	214	214	218
16	Enthalpy after assumed isentropic expansion to exhaust press., B.T.U./lb.	1046	1039	1034	1023
17	Enthalpy of saturated liquid at exhaust p. (h_{f2}), B.T.U./lb.	181.6	185.3	186.0	185.5

TABLE V. (Continued)

Item	(Run No.)	1	2	3	4
18	$(h_1 - h_{f2})$ B.T.U./lb.	977.4	983.7	985.5	980.5
19	Ideal Rankine cycle Efficiency, %	11.57	13.56	13.96	14.59
20	Duration of run, hr.	0.1667	0.1667	0.1667	0.1667
21	Output, B.T.U./hr.	1129	1403	1680	2140
22	lbs. steam per hr.	234	288	318	330
23	Output rate, B.T.U./lb.	4.82	4.88	5.28	6.18
24	Engine Efficiency, %	4.26	3.67	3.84	4.32
25	Actual Steam rate, Lb./H.P.hr.	528	523	475	393
26	Thermal Efficiency, %	0.495	0.497	0.536	0.631

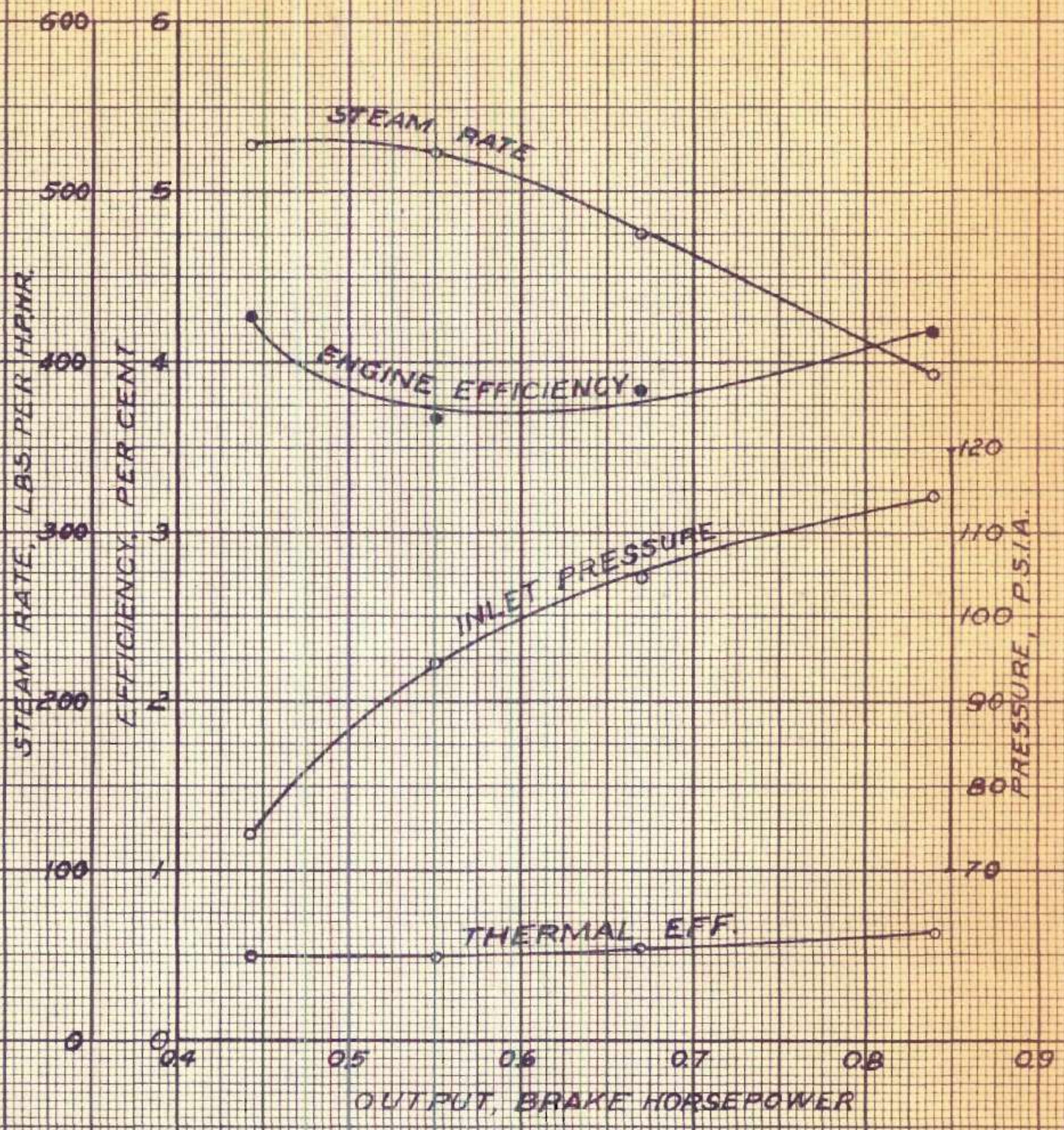


FIGURE 12. TURBINE ECONOMY RESULTS
AT 7,000 R.P.M.

VII

DISCUSSION

From the comparison of the outputs of the turbine using several different nozzles, page 29, it can be seen that both the size and shape of the nozzle have considerable influence over the resulting output. It is realized that testing the effects of only four nozzles does not determine the most efficient one. It does, however, serve to show that the type of nozzle used controls the performance in a marked way.

It was interesting to note the definite improvement in performance when the 0.269 inch diverging nozzle was used as compared with the performance when the 0.269 inch straight nozzle was used. For the same pressure drop through each of these two nozzles the steam flow should be practically the same; yet the diverging nozzle produced about one third more horsepower. A comparison between the torques at 100 p.s.i. inlet gage pressure using the different nozzles is given on page 30. The explanation for the crossing of the curves for the two 0.269 inch nozzles lies in the fact that the turbine was very unstable at low speeds, making reliable readings difficult to obtain, while at zero r.p.m. the internal friction of the turbine was hard to eliminate. Therefore, the zero r.p.m. readings must be considered as only approximate. The slow speed instability of the turbine explains why more readings in that range were not taken; they were too unreliable and difficult to obtain. At the higher speeds it was relatively easy to hold a constant r.p.m. long enough to take reliable readings.

The smaller turbines built by Tesla were all operated at extremely high speeds, between 20,000 and 30,000 r.p.m. It was suspected that the turbine built for these tests might also have to be operated at speeds approximating these values in order to obtain even moderately high efficiencies. The results of the tests do indicate that the turbine should be operated at higher speed for better performance.

A study of the results of the economy tests indicates another very interesting fact, and also helps explain further why such low efficiencies were obtained. It will be noted that the efficiency increased as the higher outputs were reached, but no peak of the efficiency curve was obtained. Similarly the steam rate was decreasing rapidly but no low point was reached. This indicates that the turbine was operating at only part of its capacity. It appears that, if further experiments were made at higher outputs using an improved dynamometer, a higher steam pressure, and, possibly, even with vacuum exhaust, much higher efficiencies would be obtained.

During the progress of the experiment several ways to improve the turbine and its performance became apparent. The ball bearings seemed to dry out after prolonged running at high speed and high temperature. To offset this trouble the turbine was stopped for a few minutes between runs to allow the oil to run back into the bearings. The ball bearings used should be replaced by bronze sleeve type bearings with a more positive oil or grease lubrication. The running temperature of the bearings could be reduced by reshaping the end bells so that the bearings would be more isolated from the turbine proper.

The nozzle should be changed from one with circular cross-section to one with rectangular cross-section so that steam could be introduced to the edges of all disks simultaneously. The single circular nozzle could direct the steam flow over only two or three of the disks. A wider rectangular nozzle or perhaps a series of circular ones could be designed so as to cover the edges of all disks simultaneously.

The size of the six one inch holes in each disk was based on an initial assumption that the steam underwent only part of its expansion in the nozzle and that it would experience appreciably further expansion in the rotor. The results of the tests showed, however, that nearly all of the expansion occurs in the nozzle and that the pressure drop in the rotor was small, being just enough to carry the steam through the turbine and out the exhausts. Therefore the holes in the disk could be appreciably reduced in size and their centers moved correspondingly nearer the center-line of the shaft, thus increasing the effective area of each rotor disk.

The conical taper given to each disk accomplishes two things. First, the strength characteristics and rotative stability of the disk are increased, and, second, the relatively sharp edges reduce turbulence as the steam is fed into the rotor.

There is one apparent drawback to the conical taper of the disks. This drawback is the severe reduction in space for the steam to flow properly near the center of the disk. It is quite possible that there was some unwanted throttling of the steam as it approached the center of the disks due to this reduction in space. This difficulty could be overcome by actually "dishing" the disks in a progressive manner such that the center disks would be nearly straight and those nearest the ends of the shaft would have a definite conical shape.

Conversations with men who have had experience with large circular saws revealed that such saws are frequently given a slight dished shape to offset the effect of higher temperatures at the periphery of the saw and add to its stability while rotating at high speed. It was, in a measure, to imitate the dished effect that the conical taper was cut on one side only of the disks of our turbine.

Still another suggested improvement would be to reduce the spacing between the disks. Possibly a combination of the progressive dishing of the disks (suggested previously) and reduced spacing at the disk edges would be the answer.

It would probably be possible to design a rotor which would produce a faster deceleration of the steam and also convert such deceleration into useful work. In figure 13 are several possible velocity-radius relationships.

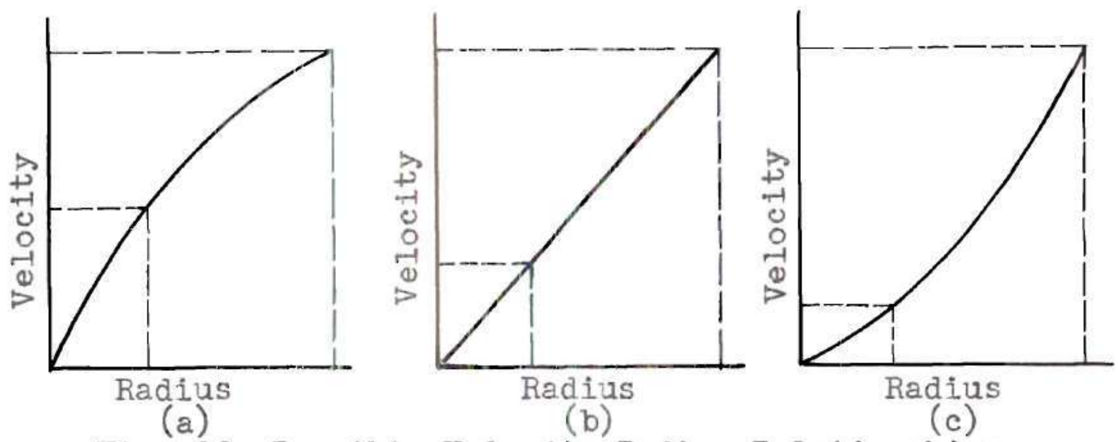


Fig. 13. Possible Velocity-Radius Relationships.

The relationship represented by (a) is probably the one most nearly representative of the conditions in the turbine which was tested. It can be seen that if a rotor were designed to cause the steam to follow the type of curve represented by (c) the larger reduction in velocity would make possible a higher output for the same entering steam velocity.

VIII

CONCLUSION

Before undertaking this investigation the writer thought that a good portion of the expansion of the steam probably occurred during the passage of the steam through the rotor of the turbine. However, it was found that nearly all of the expansion occurs in the nozzle. The turbine is then, in effect, an impulse turbine. The general characteristics of the Tesla turbine should follow those of an impulse turbine. To the extent of the completeness of the experimental results the turbine used did show such characteristics.

Tesla had suggested that two or more of his turbines be compounded in series to gain higher overall efficiencies. This does not seem to be advisable. The efficiency of the Tesla turbine seems to increase with greater pressure drops through the nozzle. With two turbines connected in series the pressure drop through either nozzle could not be as great as the drop through the nozzle of a single turbine operating with the same overall pressure drop. It seems then that the efficiency of such a combination would be rather low. It might be possible to lead the exhaust from a Tesla turbine into a low pressure reaction turbine with a possible gain in efficiency. If this were done, however, the advantage of the simplicity of the Tesla turbine would be lost, since reaction turbines are expensive, complex and rather delicate.

Even in its present state of development the Tesla turbine could be used on low power machinery, particularly where intermittent operation or reversing is required. A dairy plant which has steam available could use it to drive the separators; the exhaust could be used for heating. Other processing plants could use it in a similar manner. Chemical laboratories could use small Tesla turbines to drive centrifuges; physics laboratories could use them in a similar manner to drive small equipment at high speeds. In experiments involving the speed of light very high rotative speeds are frequently required. The Tesla turbine would be one way of obtaining these high speeds.

An interesting variation of the Tesla turbine would be to use it as a compressor. In such a case the flow would be reversed, intake being near the shaft and the discharge at the periphery.

The writer strongly recommends that further investigations be made in the hope of developing Tesla turbines which may eventually replace bladed turbines in many applications.

IX

BIBLIOGRAPHY

Gebhardt, G. F. Steam Power Plant Engineering. Fourth edition, John Wiley & Sons, 1913, p. 459.

Goldstein, S. Modern Developments in Fluid Dynamics. Oxford University Press, 1938, Volume I, p. 297, Volume II, p. 358.

Hyland, P. H. and J. B. Kommers, Machine Design, Second edition, McGraw-Hill Book Company, Inc., 1937, p. 346.

Keenan, Joseph and Frederick G. Keyes, Thermodynamic Properties of Steam. New York, John Wiley & Sons, Inc., 1936.

O'Neill, John J. Prodigal Genius, the Life of Nikola Tesla. Binghamton, N.Y., Vail-Ballou Press, Inc., 1944.

Stodola, Dr. A. Steam and Gas Turbines. Authorized translation by Dr. Louis C. Loewenstein, McGraw-Hill Book Company, Inc., 1927, Reprinted by Peter Smith, New York, 1945, two volumes.

Vennard, John K. Elementary Fluid Mechanics. New York, John Wiley & Sons, Inc., 1940.

APPENDIX I. DATA

TABLE VI. Turbine Output Data Using 0.269 Inch Straight Nozzle

Steam Pressure Lb./Sq.In. Gage	Brake Scale Pounds	Speed RPM	Date
30	0.20	0	11/12/51
30	0.13	1250	"
30	0.10	3750	"
30	0.08	4900	"
30	0.08	5500	"
40	0.21	0	"
40	0.16	2000	"
40	0.13	3750	"
40	0.10	5500	"
40	0.08	6100	"
50	0.24	0	"
50	0.20	2500	"
50	0.18	3500	"
50	0.15	5250	"
60	0.30	0	"
60	0.22	2500	"
60	0.20	4100	"
60	0.18	4700	"
60	0.16	5400	"
60	0.15	6000	"
70	0.33	0	"
70	0.28	2400	"
70	0.25	4000	"
70	0.24	5200	"
70	0.23	5350	"
70	0.20	5500	"
80	0.42	0	"
80	0.30	3300	"
80	0.28	3800	"
80	0.24	5500	"
115	0.35	6500	"

TABLE VI. (continued) Turbine Output Data Using 0.269 Inch Straight Nozzle

Steam Pressure Lb./Sq.In. Gage	Brake Scale Pounds	Speed RPM	Date
40	0.10	3700	12/3/51
40	0.05	6000	"
60	0.25	0	"
60	0.20	3000	"
60	0.16	4000	"
60	0.15	5000	"
60	0.13	5500	"
60	0.10	6150	"
60	0.10	6000	"
100	0.60	0	"
100	0.38	3000	"
100	0.36	4000	"
100	0.27	5000	"
100	0.26	6000	"

TABLE VII. Turbine Output Data Using 0.125 Inch Diverging Nozzle

Steam Pressure Lb./Sq.In. Gage	Brake Scale Pounds	Speed RPM	Date
90	0.15	0	12/18/51
90	0.02	1200	"
90	0.01	2600	"
90	0.00	4500	"
100	0.15	0	"
100	0.04	1000	"
100	0.03	1500	"
100	0.02	2900	"
100	0.01	4250	"
100	0.00	5300	"
110	0.10	0	"
110	0.04	900	"
110	0.03	2000	"
110	0.02	2200	"
110	0.01	4500	"
110	0.00	5500	"
120	0.10	0	"
120	0.15	0	"
120	0.04	1400	"
120	0.03	2000	"
120	0.02	3200	"
120	0.01	4500	"
120	0.025	3000	"

TABLE VIII. Turbine Output Data Using 0.188 Inch Throat Diverging Nozzle

Steam Pressure Lb./Sq.In. Gage	Brake Scale Pounds	Speed RPM	Date
30	0.11	0	12/19/51
30	0.02	1000	"
30	0.01	1700	"
30	0.00	2300	"
40	0.09	0	"
40	0.03	1800	"
40	0.01	3600	"
40	0.00	5500	"
50	0.15	0	"
50	0.04	2500	"
50	0.03	4250	"
50	0.02	4500	"
60	0.12	0	"
60	0.07	100	"
60	0.02	3100	"
70	0.13	0	"
70	0.05	1500	"
70	0.04	3100	"
70	0.02	4500	"
70	0.01	5800	"
80	0.18	0	"
80	0.07	3200	"
80	0.05	5000	"
80	0.03	5500	"
80	0.02	6000	"
90	0.16	0	"
90	0.07	4750	"
90	0.08	4200	"
90	0.06	5800	"
90	0.04	6250	"
90	0.03	6500	"
90	0.02	6800	"
90	0.02	7000	"

TABLE VIII. (continued) Turbine Output Data Using 0.188 Inch Throat Diverging Nozzle

Steam Pressure Lb./Sq.In. Gage	Brake Scale Pounds	Speed RPM	Date
100	0.10	6000	12/19/51
100	0.08	6500	"
100	0.12	4800	"
100	0.30	0	"
110	0.27	0	"
110	0.15	3500	"
110	0.13	5000	"
110	0.11	6800	"
120	0.33	0	"
120	0.17	5000	"
120	0.16	5400	"
120	0.15	6200	"
120	0.14	6800	"
130	0.19	6600	"
130	0.17	7000	"
130	0.20	5500	"
130	0.25	3000	"
130	0.37	0	"
140	0.37	0	"
140	0.24	5700	"
140	0.20	6000	"

TABLE IX. Turbine Output Data Using 0.269 Inch Diverging Nozzle

Steam Pressure Lb./Sq.In. Gage	Brake Scale Pounds	Speed RPM	Turbine Pressure	Exhaust Pressure	Date
40	0.25	0	0.4	0.2	1/7/51
40	0.10	4000	0.4	0.2	"
40	0.08	4800	0.4	0.2	"
40	0.06	5700	0.4	0.2	"
40	0.05	6500	0.4	0.2	"
60	0.30	0	1.2	0.6	"
60	0.20	5000	1.4	0.7	"
60	0.20	5300	1.6	0.8	"
60	0.19	5900	1.8	0.9	"
60	0.15	7000	1.8	0.9	"
80	0.40	0	2.0	1.0	"
80	0.30	4600	2.4	0.9	"
80	0.25	6500	2.4	0.9	"
100	0.57	0	2.4	1.2	"
100	0.45	3300	2.4	1.2	"
100	0.40	4800	2.4	1.2	"
100	0.37	6200	2.4	1.3	"
118	0.60	0	4.0	1.7	"
118	0.50	5700	4.0	1.7	"
118	0.48	6200	4.0	1.7	"

TABLE X. Turbine Output Data Using 0.269 Diverging Nozzle and Unbalanced Brake

Steam Pressure Lb./Sq.In. Gage	Brake Scale Pounds	Speed RPM	Date
100	0.65	5000	1/14/52
100	0.60	6250	"
100	0.55	7500	"
100	0.50	7900	"
100	0.77	0	"
118	0.75	4500	"
118	0.75	5000	"
118	0.70	6800	"
118	0.65	8300	"

Brake Tare = 0.25 lbs.

TABLE XI. Turbine Economy Data

Steam Pressure	Brake Scale	Speed	Pressures		Temperatures		Time	Water
p.s.i. Gage	Pounds	RPM	Turb. p.s.i. Gage	Exh. p.s.i. Gage	Calor. °F	Exh. °F		Lbs.
60	0.19	7000	1.6	0.8	230	213	5:35	86.0
60	0.19	7000	1.6	0.8	230	213		
60	0.19	7000	1.6	0.8	230	213		
60	0.19	7000	1.6	0.8	230	213	5:45	125.0
80	0.22	7000	1.8	0.9	249	214	5:00	86.0
80	0.24	7000	1.8	0.9	249	214		
80	0.25	7000	1.8	0.9	250	214		
80	0.25	7000	1.8	0.9	250	214		
80	0.22	7000	1.8	0.9	250	214	5:10	134.0
90	0.27	7000	2.2	1.1	256	214	5:20	86.0
90	0.27	7000	2.2	1.1	256	214		
90	0.30	7000	2.2	1.1	256	214		
90	0.29	7000	2.2	1.1	256	214		
90	0.29	7000	2.2	1.1	256	214		
90	0.30	7000	2.2	1.1	256	214	5:30	139.0

Date 1/23/52 Barometer = 29.24" Hg

0.269 inch diverging nozzle

TABLE XII. Turbine Economy Data

Steam Pressure	Brake Scale	Speed	Pressures		Temperatures		Time	Water
p.s.i. Gage	Pounds	RPM	Turb. p.s.i. Gage	Exh. p.s.i. Gage	Calor. ^oF	Exh. ^oF		Lbs.
100	0.33	7000	2.4	1.1	244	218	4:43	87.0
100	0.35	7000	2.4	1.1	244	218		
100	0.38	7000	2.4	1.1	244	218		
100	0.38	7000	2.4	1.1	244	218	4:53	142.0

Date 2/5/52 Barometer = 28.88" Hg

0.269 inch diverging nozzle

APPENDIX II

SAMPLE CALCULATIONS

Table I.

Refer to data Table VI, pages 42 and 43.

1. Press. = 100 p.s.i. gage, for example

2. Speed = 6000 r.p.m., for example

3. Torque = scale reading, lbs. x $\frac{21 \text{ in.}}{12 \text{ in./ft.}}$

$$= 0.26 \text{ lbs.} \times \frac{21 \text{ ft.}}{12}$$

$$= 0.455 \text{ lb. ft.}$$

4. Brake H.P. = $\frac{\text{Scale reading x Speed}}{3000}$

$$= \frac{0.26 \times 6000}{3000}$$

$$= 0.52 \text{ H.P.}$$

Table II.
Table III.
Table IV.

Calculations same as for Table I.

Table V.

Refer to data Tables XI and XII, pages 49 and 50.

1. Run no. 1, for example

2. Inlet press. = 60 p.s.i. gage

3. Speed = 7000 r.p.m.

4. Brake scale = 0.190 lbs.

5. Brake H.P. = $\frac{\text{Brake scale x Speed}}{3000}$

Table V. cont'd.

5. (cont'd.)

$$= \frac{0.190 \times 7,000}{3,000}$$
$$= 0.443 \text{ H.P.}$$

6. Torque = Brake scale, lbs. x $\frac{21}{12}$ ft.

$$= 0.190 \times \frac{21}{12}$$
$$= 0.333 \text{ lb./ft.}$$

7. Barometric pressure = 29.24 in.Hg.

8. Barometric

press.,p.s.i. = 29.24 in.Hg. x 0.491 $\frac{\text{p.s.i.}}{\text{in.Hg.}}$

= 14.36 p.s.i.

9. Inlet press. = Gage reading + barometric press.

= 60.6 + 14.36

= 74.36 p.s.i. abs.

10. Calorimeter temperature = 230 $^{\circ}$F

11. Quality of steam:

(a) Locate point on Mollier chart where calorimeter temperature and barometric pressure cross. Note enthalpy.

(b) At constant enthalpy trace across Mollier chart to inlet pressure, absolute. Read percent moisture.

% moisture = 2.6%

Quality = 100.0% - % moisture

= 100.0% - 2.6%

= 97.4%

Table V. cont'd.

12. Enthalpy of entering steam, noted in Item 11.

$$h_1 = 1159.0 \text{ B.T.U./lb.}$$

13. Exhaust pressure = 0.8 p.s.i. gage.

14. Exhaust pressure = 0.8 + 14.36

$$= 15.16 \text{ p.s.i. abs.}$$

15. Exhaust temp. = 213°F

16. Enthalpy after assumed isentropic expansion to exhaust pressure. Trace isentropic expansion on Mollier chart:

$$h_2' = 1046$$

17. Enthalpy of saturated liquid at exhaust pressure.

    (a) Exh.press. = 15.16 p.s.i. abs.

    (b) From steam tables:

    for 16.0 p.s.i., $h_f = 184.42$ B.T.U./lb.

    for 15.0 p.s.i., $h_f = 181.11$ B.T.U./lb.

$$\Delta(h_f) = 184.42 - 181.11$$

$$= 3.31 \text{ B.T.U./lb.}$$

    (c) $3.31 \times \frac{0.16}{1.00} = 0.530$ B.T.U./lb.

    (d) $(h_{f2}) = 181.11 + 0.53$

$$= 181.64 \text{ B.T.U./lb.}$$

18. $h_1 - h_{f2} = 1159.0 - 181.64$

$$= 977.36 \text{ B.T.U./lb.}$$

19. Ideal Rankine cycle efficiency

$$= \frac{h_1 - h_2'}{h_1 - h_{f2}} \; 100\%$$

Table V. cont'd.

19. (cont'd.)

$$= \left(\frac{1159.0 - 1046.0}{977.36}\right) 100\%$$

$$= 11.57\%$$

20. Duration of run = 10 minutes

$$= 0.1667 \text{ hours}$$

21. Output B.T.U./hr. = H.P. x 2545 B.T.U./H.P.hr.

$$= 0.443 \times 2545$$

$$= 1129 \text{ B.T.U./hr.}$$

22. Lbs. steam per hr. $= \frac{\text{lbs. steam per run}}{\text{Length of run}}$

$$= \frac{(125.0 - 86.0) \text{ lbs.}}{0.1667 \text{ hr.}}$$

$$= 234 \text{ lbs./hr.}$$

23. Output rate $= \frac{\text{Output, B.T.U./hr.}}{\text{Lbs. Steam/hr.}}$

$$= \frac{1129}{234}$$

$$= 4.82 \text{ B.T.U./lb.}$$

24. Engine efficiency $= \frac{\text{Output rate}}{h_1 - h_2} \times 100\%$

$$= \frac{4.82}{1159.0 - 1046.0} \times 100\%$$

$$= 4.26\%$$

25. Actual Steam rate $= \frac{\text{lbs. steam per hr.}}{\text{H.P.}}$

$$= \frac{234}{0.443}$$

$$= 528 \text{ lbs./H.P.hr.}$$

26. Thermal efficiency $= \frac{\text{Output rate}}{h_1 - h_{f2}} \times 100\%$

$$= \frac{4.26}{977.4} \times 100\%$$

$$= 0.495\%$$

APPENDIX III

CALCULATION OF THE THEORETICAL OUTPUT

An attempt will now be made to derive an equation for the theoretical output of the turbine.

Considering one face of a single disk an expression for the power output can be written in differential form. The y direction will be chosen perpendicular to the plane of the disk.

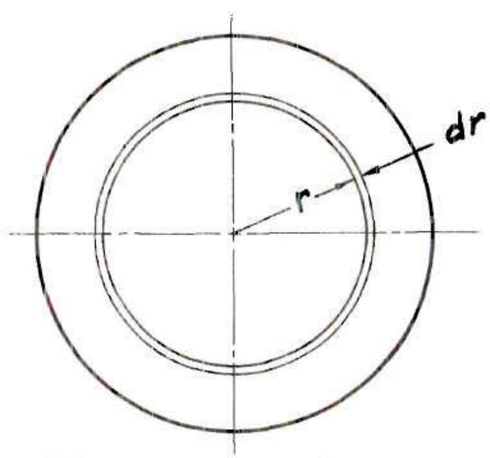


Fig. 14. Differential Element of Area for Turbine Disk

The shearing stress in the laminar boundary layer due to the motion of the steam around the face of the disk is[5]

$$\tau = \mu \frac{dV}{dy}$$

---

[5]John K. Vennard, Elementary Fluid Mechanics, New York, John Wiley & Sons, Inc., 1940, p. 158.

This can be written as approximately

$$\tau = \mu \frac{\Delta V}{\delta} \tag{1}$$

The element of area upon which this stress is acting is:

$$dA = 2\pi r dr \tag{2}$$

Multiplying the stress by the element of area gives the element of force, and further multiplication by r gives the moment of the force, or the element of torque, about the axis of the disk.

$$dT = \frac{2\pi\mu(\Delta V)r^2 dr}{\delta} \tag{3}$$

Introducing the speed of revolution, n, and the proper coefficients[6] gives an expression for power.

$$dP = \left(\frac{2\pi n}{33,000}\right) \frac{2\pi\mu(\Delta V)r^2 dr}{\delta}$$

Collecting the coefficients:

$$dP = \frac{(1.20 \times 10^{-3})\ n\mu(\Delta V)r^2 dr}{\delta} \tag{4}$$

This is a general expression and is good for any Tesla turbine disk face. However, (ΔV), μ, and δ must be expressed as functions of the radius before this equation can be integrated. These quantities all depend upon the

---

[6] P. H. Hyland and J. B. Kommers, Machine Design, Second edition, McGraw-Hill Book Company, Inc., 1937, p. 346.

characteristics of the turbine itself.

ΔV may be expressed as

$$\Delta V = V - \frac{2\pi nr}{60}$$

$$\Delta V = V - 0.1047nr \tag{5}$$

It is necessary to know the relationship between the radius from the center line of the turbine shaft and the steam velocity. Since torque is being developed, the equations for a free vortex cannot be used. However, the general relationship[7]

$$\frac{V^2}{r} = \frac{dp}{\rho\, dr} \tag{6}$$

for fluid moving in a curved path is valid. The variation of pressure through the rotor disks is difficult to determine exactly, however the pressures at the inside of the housing and at the exhaust can be measured. A plot of these pressures is given on page 58. The assumption was made that the pressure at 0.1 ft. radius was equal to the exhaust pressure. A correction could have been assumed but there is no assurance that the correction would be any better than the assumption just mentioned.

It can be seen that though there is a pressure drop

---

[7]John K. Vennard, Elementary Fluid Mechanics, p. 93.

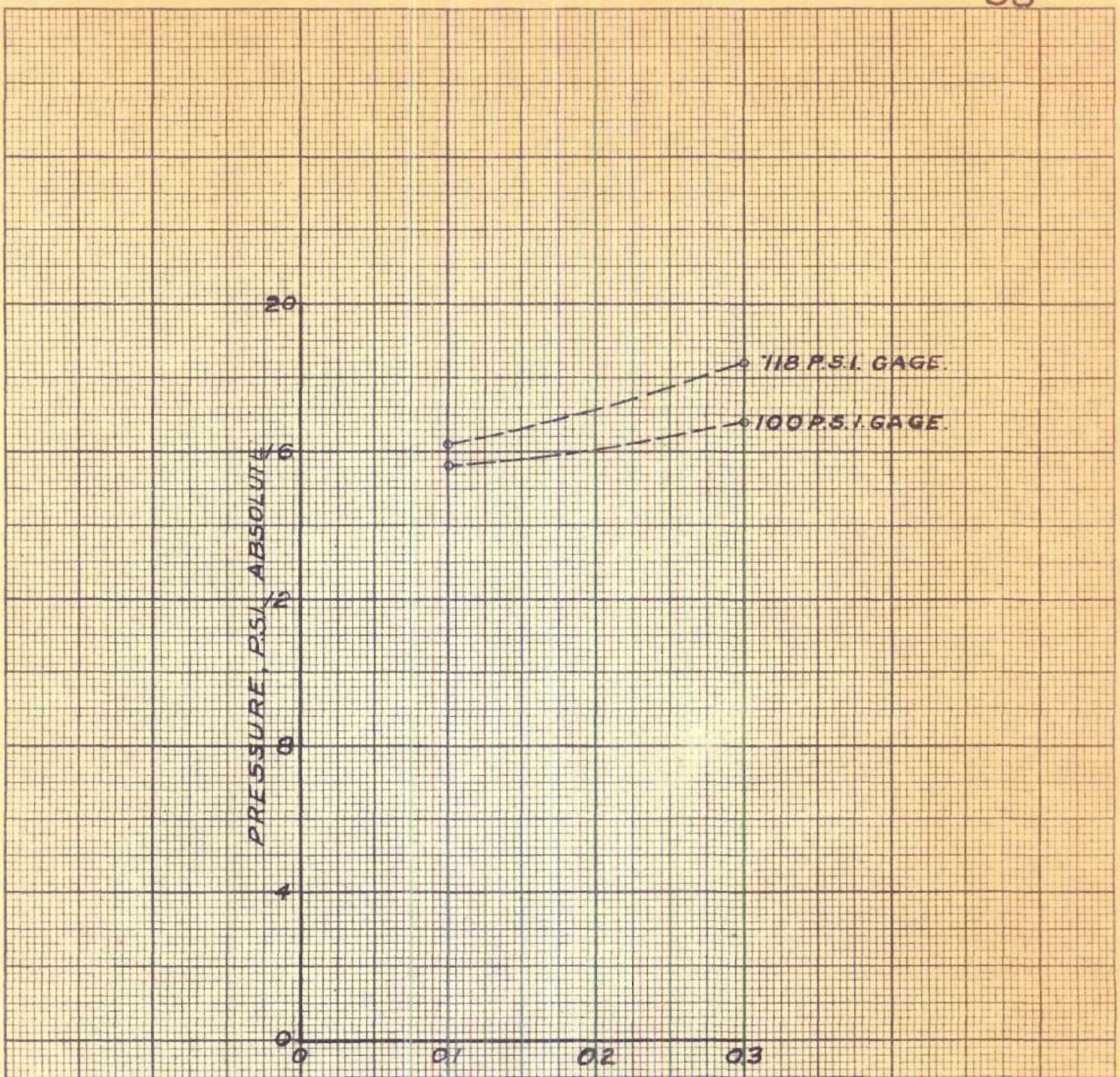


FIG. 15. PRESSURE DROP ACROSS ROTOR DISKS.
FOR TWO ENTRANCE PRESSURES

the drop is not great, being just over 2 p.s.i. during the highest run. The curves joining these points will have to be estimated. There are three general possibilities for the shape of these curves. They are: concave downward, concave upward, or straight. This pressure-radius relation for a free vortex is concave downward; for a completely forced vortex it is concave upward. For this turbine there is definitely a combination of the two types of vortices. Since appreciable torque is exerted on the moving steam, it is reasonable to assume that the p-r curve is concave upward. In such a case:

$$\frac{dp}{dr} = k'r^{n'} + c'$$

where k' and c' are constants and n is greater than zero. The density, ρ , also decreases with decreasing radius:

$$\rho = k''r^{n''} + c''$$

In view of the fact that both $\frac{dp}{dr}$ and ρ actually change but little between the values where r = 0.3 ft. and r = 0.1 ft., it is not too unreasonable to assume that the ratio

$$\frac{k'r^{n'} + c'}{k''r^{n''} + c''}$$

is almost constant. Calling this constant K^2 we then have:

$$\frac{V^2}{r} = K^2$$

or

$$V = K\sqrt{r} \tag{7}$$

Then

$$K = \frac{V_1}{\sqrt{r_1}} \qquad = \frac{V_2}{\sqrt{r_2}} \tag{8}$$

In order to determine δ it is first necessary to determine the Reynolds number and the friction factor.

For fluid flowing between parallel (or nearly parallel) plates the Reynolds number is defined as[8]

$$R = \frac{(\Delta V)\ \rho\ (4m)}{\mu} \tag{9}$$

where

m = cross sectional area ÷ wetted perimeter.

For parallel plates

$$m = \frac{d}{2}$$

where d is the distance between plates. The Reynolds number for parallel plates then becomes:

$$R = \frac{(\Delta V)\ \rho\ (2d)}{\mu} \tag{9a}$$

---

[8] S. Goldstein, Modern Developments in Fluid Dynamics, Oxford University Press, 1938, Volume I, p. 297.

For a circular pipe the Reynolds number is defined as:[9]

$$R = \frac{(\Delta V)\rho(4m)}{\mu} \tag{9}$$

where

$$m = \frac{\pi r^2}{2\pi r}$$
$$= \frac{r}{2}$$

Then

$$4m = 2r$$
$$= \text{diameter of pipe}$$

or

$$R = \frac{(\Delta V)\rho(\text{diam.})}{\mu} \tag{9b}$$

which is the form used in Vennard for circular pipes.

Goldstein[10] states that, when (4m) is used to define the Reynolds number, the results obtained for flow between parallel plates do not lie far from those obtained for flow in circular pipes. On this basis one is justified in using

[9] S. Goldstein, Modern Developments in Fluid Dynamics, Volume I, p. 297.

[10] S. Goldstein, Modern Developments in Fluid Dynamics, Volume II, p. 358.

the friction factor table in Vennard,[11] provided the following relation is kept in mind and adhered to:

$$(4m)\ \text{pipe} = (4m)\ \text{plates}$$

or

$$(\text{diam.})\ \text{pipe} = (2d)\ \text{plates} \qquad (10)$$

The thickness of the laminar film, δ, can now be computed using an adaptation of this formula:[12]

$$\frac{\delta}{\text{pipe dia.}} = \frac{32.8}{R\sqrt{f}}$$

Substitution of eq. 10 for the pipe diameter gives:

$$\delta = \frac{32.8 \times 2d}{R \quad \sqrt{f}} \qquad (11)$$

Substituting for the Reynolds number, eq. 9a,

$$\delta = \frac{32.8\ (2d)\ \mu}{(\Delta v)\rho\,(2d)\sqrt{f}}$$

$$= \frac{32.8\ \mu}{(\Delta v)\rho\ \sqrt{f}}$$

---

[11]John K. Vennard, Elementary Fluid Mechanics, p. 153.

[12]John K. Vennard, Elementary Fluid Mechanics, p. 159.

Substituting this expression for δ into eq. 4:

$$dP = \frac{(1.20x10^{-3})n\mu(\Delta V)r^2dr}{\frac{32.8\ \mu}{(\Delta V)\rho\sqrt{f}}}$$

$$dP = (3.66x10^{-5})n\rho\sqrt{f}(\Delta V)^2r^2dr \tag{4a}$$

Substituting for (ΔV), eq. 5:

$$dP = (3.66x10^{-5})n\rho\sqrt{f}(V-0.1047nr)^2r^2dr$$

$$dP = (3.66x10^{-5})n\rho\sqrt{f}(V^2r^2-0.2094Vnr^3 + 0.01095n^2r^4)dr$$

From eq. 7:

$$V = K\sqrt{r} \tag{7}$$

Then

$$dP = (3.66x10^{-5})n\rho\sqrt{f}(K^2r^3-0.2094Knr^{7/2}+0.01095n^2r^4)dr \tag{4b}$$

An assumption that the density, ρ, and the friction factor, f, are constant will now be made. For the turbine used, the product $\rho\sqrt{f}$ is actually very nearly constant. Integration of eq. 4b gives:

$$P = (3.66x10^{-5})n\rho\sqrt{f}\left[\frac{K^2r^4}{4}-\frac{0.4188Knr^{9/2}}{9}+\frac{0.01095n^2r^5}{5}\right]_{r_1}^{r_2} \tag{12}$$

This is an expression for the power developed by one surface of a disk in a Tesla turbine.

To compare this equation with the actual output of the turbine tested the conditions for one of the higher output runs will be chosen:

$$p_1 = 134.4 \text{ p.s.i.a. (assume saturated)}$$
$$p_2 = 18.4 \text{ p.s.i.a.}$$

From steam tables and Mollier diagram[13]

$$v = 19.25 \text{ cu. ft. per lb.}$$

The mass density is obtained as follows:

$$\rho = \frac{1}{19.25} \times \frac{1}{32.2}$$
$$= 1.62 \times 10^{-3} \text{ slugs per cu. ft.} \qquad (13)$$

From the chart in Vennard[14] the viscosity, μ, for wet steam at 15 p.s.i.a. is found to be:

$$\mu = 3.1 \times 10^{-7} \text{ lb. sec./sq.ft.} \qquad (14)$$

The average distance between the disks of the turbine is about 0.2 inches, allowing for the greater area on the outer portion of the disks. Then:

$$2d_{ave.} = 0.4 \text{ inches}$$

---

[13]Joseph H. Keenan and Frederick G. Keyes, <u>Thermodynamic Properties of Steam</u>, New York, John Wiley & Sins, Inc., 1936.

[14]John K. Vennard, <u>Elementary Fluid Mechanics</u>, p. 12.

$$2d_{ave.} = 0.033 \text{ feet}$$

As this is just slightly above the inside diameter of a $\frac{1}{4}$ inch standard steel pipe, the friction factor based on $\frac{1}{4}$ inch steel pipe will be used. In order to use Vennard's friction factor table it is necessary to evaluate the Reynolds number. In the data chosen previously the pressures were:

$$p_1 = 134.4 \text{ p.s.i.a. (assume saturated)}$$
$$p_2 = 18.4 \text{ p.s.i.a.}$$

Then from the Mollier Chart[15]

$$h_1 = 1192 \text{ B.T.U./lb.}$$
$$h_2 = 1045 \text{ B.T.U./lb. (assuming reversible adiabatic expansion)}$$

then

$$\begin{aligned} V_1 &= \sqrt{2gJ\ (h_1 - h_2)} \\ &= \sqrt{2x32.2x778\ (1192-1045)} \\ &= 2710 \text{ ft./sec.} \end{aligned} \tag{7a}$$

To solve for K (eq. 8):

$$K = \frac{2710}{\sqrt{0.292}}$$

---

[15]Joseph H. Keenan and Frederick G. Keyes, Thermodynamic Properties of Steam, Mollier Chart insert.

$$K = 4590 \text{ ft}^{\frac{1}{2}}/\text{sec.} \tag{8a}$$

At the inner effective radius of the disks, 0.10 ft.,

$$\begin{aligned} V_2 &= 4590 \sqrt{0.10} \\ &= 1450 \text{ ft/sec.} \end{aligned} \tag{7b}$$

Based on these values of velocity an average value for ΔV of 2200 feet per second will be chosen. The Reynolds number then becomes:

$$\begin{aligned} R &= \frac{2200 \times 0.033 \times 0.0521}{3.1 \times 10^{-7} \times 32.2} \\ &= 3.75 \times 10^5 \end{aligned} \tag{9c}$$

The friction factor may now be read from the table in Vennard[16]

$$f = 0.030 \tag{15}$$

Substitution of these specific values for ρ and f into eq. 12 gives:

$$P = 1.03(10^{-8})n \left[\frac{K^2 r^4}{4} - \frac{0.4188Knr^{9/2}}{9} + \frac{0.01095n^2 r^5}{5} \right]_{r_1}^{r_2} \tag{12a}$$

The limits are:

$$r_2 = 0.292 \text{ ft.}$$

---

[16] John K. Vennard, Elementary Fluid Mechanics, p. 153.

$$r_1 = 0.100 \text{ ft.}$$

Substituting the limits:

$$P = 1.03(10^{-8})n\left[0.0018K^2-0.000181Kn+0.00000458n^2\right] \qquad (12b)$$

The bracket is very close to a perfect square, so the equation becomes:

$$P = 1.03(10^{-8})n\left[0.042K-0.00214n\right]^2 \qquad (12c)$$

For the assumed conditions of this analysis:

$$K = 4590 \text{ ft.}^{\frac{1}{2}}\text{per sec.} \qquad (8a)$$

Then

$$P = 1.03(10^{-8})n\left[195-0.00214n\right]^2 \qquad (12d)$$

which is the power developed by one side of one disk. For 20 disk sides:

$$P = 20.6(10^{-8})n\left[195-0.00214n\right]^2 \qquad (16)$$

To determine the speed for maximum power, differentiate eq. 12b with respect to n:

$$\frac{dP}{dn} = 1.03(10^{-8})\left[0.0018K^2-0.000362Kn+0.0000137n^2\right] \qquad (17)$$

Setting the derivative equal to zero and substituting for K, n has two values:

$$n = 30{,}500 \text{ r.p.m.}$$

$$n = 90{,}700 \text{ r.p.m.}$$

Substitution of 90,700 r.p.m. for n into the output equation gives zero H.P., therefore the true maximum output occurs at 30,500 r.p.m. At this speed, with 20 disk surfaces, the theoretical output would be:

$$P = 105.8 \text{ H.P.}$$

In deriving these equations no consideration was given to the quantity of steam required to produce this power. Considering reversible adiabatic expansion of the steam and conversion of all the energy into work, the output for the conditions used in the derivation would be:

$$\begin{aligned} h_1 - h_2 &= 1192 - 1045 \\ &= 147 \text{ B.T.U. per lb. of steam} \end{aligned}$$

From the curves on page 28 the power output at 7000 r.p.m. and 120 p.s.i. gage pressure would be about 1.05 horsepower. Extrapolation of the curves on page 33 indicates that for 1.05 horsepower at 7000 r.p.m. the steam required is about 300 pounds per hour.

For the assumed pressure limits the maximum output from 300 lbs. of steam per hour would be:

$$\begin{aligned} P &= \frac{300 \times 147}{2545} \\ &= 17.3 \text{ H.P.} \end{aligned}$$

At 7000 r.p.m. the equation derived for the turbine power gives an output of:

$$P = 46.7 \text{ H.P.}$$

Thus it is seen a much larger nozzle is needed for the turbine to give maximum performance. A nozzle with about three times the capacity of the 0.269 inch diverging nozzle would be suggested, or, perhaps, three 0.269 inch diverging nozzles in parallel.

At 7000 r.p.m. there was no way of determining the friction horsepower lost in the bearings, but this loss could easily have been on the order of one horsepower. This is enough loss to seriously affect the apparent efficiency of the turbine.

It must be pointed out that the derived equation is for indicated horsepower, and not brake horsepower, though, if friction losses were small the two should approach each other in value.

Since there is such a large discrepancy between the derived equation and the performance of the turbine, the equation must not be considered correct. The writer plans to continue his investigations in the hope of improving the turbine and of deriving a more nearly correct equation.

APPENDIX IV

LIST OF SYMBOLS

Symbol	Description
A	= Area, square feet.
d	= Distance between adjacent disk surfaces, feet.
Note:	Where the term diameter is intended, it is spelled out or abbreviated as dia.
f	= Friction factor.
g	= 32.2 ft./sec.^{2}
h	= Enthalpy, B.T.U. per pound.
J	= 778 ft.lb./B.T.U.
K, k	= Constants explained in Appendix III.
m	= Hydraulic radius = Cross-sectional area ÷ wetted perimeter, feet.
n	= Rotor speed, revolutions per minute.
P	= Power, units of horsepower.
p	= Pressure, pounds per square inch.
R	= Reynolds number.
r	= Radius, feet.
T	= Torque, pounds feet.
V	= Velocity of steam, feet per second.
v	= Specific volume, cubic feet per pound.
δ	= Thickness of laminar boundary layer, feet.
μ	= Viscosity, pound seconds per square foot.
ρ	= Mass density, slugs per cubic foot.
τ	= Shearing stress in the laminar boundary layer of the fluid, pounds per square foot.